

Mathematics applications

Unit (1)

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UNIT ONE: STATICS

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Lesson (1): Forces

The force

"The force is defined as the effect of a natural body upon another one" by pushing, attraction, pressure or repulsion".

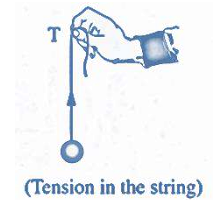
The natural body is a body consisting of material (mass) and volume not equal to zero,

Kinds of forces

There are different kinds of forces? as:

1) Tension force (T):

As the force in the string (or the rope) when carrying a body at it.

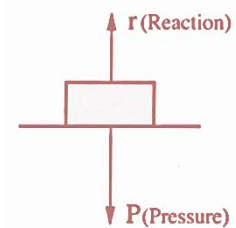


2) Pressure force (P):

As the force that appears when a body stabilizes on a surface.

3) Reaction force (r):

As the reaction of a smooth surface on a body stabilized on it.



4) Attraction forces and repulsion forces:

As the forces which formed between magnetic poles, electric charges and astronomical objects.

5) Gravitational forces (weights):

If we let a body in the air, then it will drop down towards the Earth because the attraction force of the Earth attracts anybody towards it.

This force is called Gravitational force or weights.

Notice that: the weight (W) = the body mass x acceleration of gravity = $m \times g$

The force can be expressed as follow:

- (1) $\vec{F} = (X, y)$ $\Rightarrow$ the cartesian form .
- (2) $\vec{F} = X \vec{i} + y \vec{j}$ $\Rightarrow$ in term s of the fundamental unit vectors.
- (3) $\vec{F} = (|| \vec{F} || , \theta) \Rightarrow$ The polar form .

The measurement units:

- 1) 1 kg.wt. = 1000 gm.wt. = 103 gm.wt.
- 2) 1 newton = 100 000 dyne - 105 dyne
- 3) 1 kg.wt. = 9.8 newton , 1 gm.wt. = 980 dyne

Main Points

⇒ **Statics:** The branch of mechanics that deals with bodies at rest or forces in equilibrium.

⇒ **Force:** The effect of a natural body upon another one.

⇒ **Properties of a force:** The effect of a force is determined by:

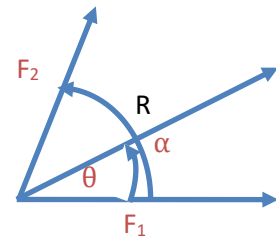
- 1- Magnitude. 2- Direction. 3- Point of action. (Line of action)

If $\vec{F}_1$, $\vec{F}_2$ are two forces , the angle between their lines of actions (α) , their resultant $\vec{R}$ which inclined by an angle of measure (θ) with the direction of $\vec{F}_1$ then:

&

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha} \quad (\text{Magnitude})$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \quad (\text{Direction})$$

Special cases:

i) If $F_1 \perp F_2 \Rightarrow$

$$R = \sqrt{F_1^2 + F_2^2}$$

, and

$$\tan \theta = \frac{F_2}{F_1}$$

ii) If $F_1 = F_2 = F \Rightarrow \theta = \frac{\alpha}{2}$ and R bisects the angle between F_1 and F_2 ,

$$R = 2F \cos \frac{\alpha}{2}$$

iii) if $\alpha = 0$ then $R = F_1 + F_2$ (maximum resultant)

iv) if $\alpha = 180$ then $R = |F_1 - F_2|$ (minimum resultant)

v) If $\theta = 90^\circ \Rightarrow F_1 + F_2 \cos \alpha = 0$ and $R = \sqrt{F_2^2 - F_1^2}$

Note: If we want the measure of the angle between R and F_2 , Find $(\alpha - \theta)$

Or from the law:

$$\tan \theta = \frac{F_1 \sin \alpha}{F_2 + F_1 \cos \alpha}$$

1) Two forces of magnitudes 5 newton and 3 newton act at a point and include an angle of measure 60° ? find their resultant in magnitude and direction analytically.

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha}$$

$$R = \sqrt{(5)^2 + (3)^2 + 2 \times 5 \times 3 \times \cos 60} = 7 \text{ newton.}$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\tan \theta = \frac{3 \sin 60}{5 + 3 \cos 60} = \frac{3\sqrt{3}}{13}$$

$$\theta = 21^\circ 47'$$

The magnitude of R is 7 newton and include an angle of measure $21^\circ 47'$ with the first force.

2) Two perpendicular forces act at a point such that $F_1 = 6$ newton and $F_2 = 2.5$ newton. Find their resultant in magnitude and find its direction.

$$R = \sqrt{F_1^2 + F_2^2}$$

$$R = \sqrt{6^2 + 2.5^2} = 6.5 \text{ newton.}$$

$$\tan \theta = \frac{F_2}{F_1}$$

$$\tan \theta = \frac{2.5}{6} = \frac{5}{12} \quad \theta = 22^\circ 37'$$

The magnitude of R is 6.5 newton and include an angle of measure $22^\circ 37'$ with the first force.

3) Two forces of magnitudes 50 newton and 100 newton act at a point. Their resultant is perpendicular to the first force. Find the measure of the angle included between the two forces and the magnitude of the resultant.

$F_1 = 50$ newton , $F_2 = 100$ newton.

The resultant is perpendicular to the first force.

$$F_1 + F_2 \cos \alpha = 0$$

$$50 + 100 \cos \alpha = 0$$

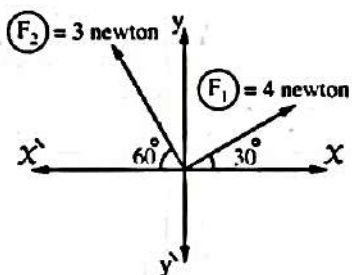
$$\cos \alpha = \frac{-1}{2}$$

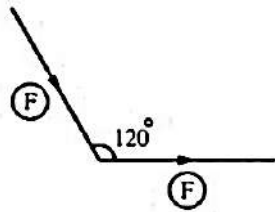
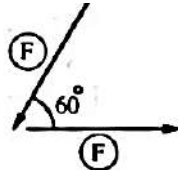
$$\alpha = 120^\circ$$

$$R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos \alpha}$$

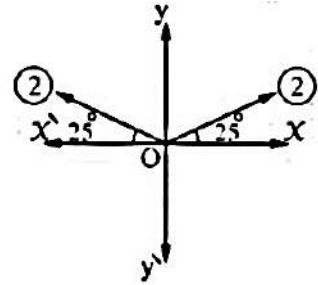
$$R = \sqrt{(50)^2 + (100)^2 + 2 \times 50 \times 100 \times \cos 120} = 50\sqrt{3} \text{ newton.}$$

[1] Choose the correct answer:

1)	The force is defined by	
	(a) its magnitude. (b) its direction. (c) the point of action. (d) all the previous.	
2)	Two forces act at a point. The magnitude of the two forces are 5 , 3 newton and the angle between them 60° , then the magnitude of their resultant = newton. (a) 2 (b) 5 (c) 7 (d) 8	
3)	Two perpendicular forces act at a point. The magnitude of the two forces 12 , 5 newton , then the magnitude of their resultant = newton. (a) 17 (b) 7 (c) 13 (d) 14	
4)	Resultant of two forces 6 newton and 8 newton could be newton. (a) 20 (b) 15 (c) 12 (d) 1	
5)	The magnitude of two forces are 4 , 5 N. They act at a point and cosine of their included angle is $\frac{-2}{5}$, then the magnitude of their resultant R = newtons. (a) 15 (b) 5 (c) 20 (d) 25	
6)	Two forces act at a point. The magnitude of the two forces are 6 , 3 newton and their resultant is perpendicular to one of them , then the magnitude of their resultant = newton. (a) 3 (b) $3\sqrt{3}$ (c) 6 (d) $6\sqrt{3}$	
7)	In the opposite figure : The magnitude of the resultant of the two forces in the figure = newton. (a) 7 (b) 5 (c) 1 (d) $\sqrt{7}$	

8)	In the opposite figure : Magnitude of the resultant of the two forces = newton. (a) $2F$ (b) F (c) $\sqrt{3}F$ (d) zero	
9)	The magnitude of the resultant of the two forces shown in the opposite figure is (a) $\frac{1}{2}F$ (b) F (c) $\sqrt{3}F$ (d) $\sqrt{5}F$	
10)	Two forces act at a point. The magnitude of the two forces are F , 2 newton and the measure of the angle between them is 60° , if their resultant equal $2\sqrt{3}$ newton , then $F = \dots\dots\dots$ newton. (a) 2 (b) 4 (c) 8 (d) 12	
11)	The magnitude of two forces F , 2 newton and the measure of their included angle $= \frac{2\pi}{3}$ and the magnitude of their resultant is F newton , then $F = \dots\dots\dots$ newton. (a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$	
12)	 Two forces of equal magnitudes , enclosing between them an angle of measure $\frac{\pi}{2}$ If the magnitude of their resultant is 8 N. , then the value of each force measured in newton is (a) $2\sqrt{2}$ (b) 4 (c) $4\sqrt{2}$ (d) 8	
13)	The magnitude of two forces F , F kg.wt. , the magnitude of their resultant 24 kg.wt. and inclined to the first force by an angle of measure 30° , then $F = \dots\dots\dots$ kg.wt. (a) 8 (b) $8\sqrt{3}$ (c) $8\sqrt{2}$ (d) 12	

14)	Two forces of magnitudes 8 and F gm.wt. The measure of the angle between them is $\alpha \in]0, \pi[$, their resultant bisects the included angle between them, then F = gm.wt. (a) 4 (b) 16 (c) $2\sqrt{2}$ (d) 8
15)	 Two forces of magnitudes 3, F newton and the measure of the angle between them is 120° . If their resultant is perpendicular to the first force, so the value of F in newton is (a) 1.5 (b) 3 (c) $3\sqrt{3}$ (d) 6
16)	 Two equal forces, the magnitude of each of them is 6 N., the magnitude of their resultant is 6 N., then the angle between them equals (a) 30° (b) 60° (c) 120° (d) 150°
17)	Two forces of magnitudes 6 N. and 8 N., if the magnitude of their resultant is 2 N., then the measure of the angle between the two forces is (a) 30° (b) 90° (c) 180° (d) 270°
18)	Two forces of magnitudes 3 F and F newton and their resultant is 4 F newton, then the measure of the angle between them = (a) 60° (b) 0° (c) 180° (d) 90°
19)	Two forces act at a point. The magnitude of the two forces are 5 F, 3 F. If the maximum value of their resultant is 40 newton, then the minimum value of their resultant newton. (a) 10 (b) 20 (c) 5 (d) zero
20)	Two forces act at a point. The magnitudes of the two forces are 5, 3 newton, then the magnitude of their resultant measure by newton $\in$ (a) $[2, 8]$ (b) $]2, 8[$ (c) $[3, 5]$ (d) $]3, 5[$

21)	The magnitude of two forces are 12 , 17 newton then the difference between the greatest and the smallest value of their resultant = newton. (a) 29 (b) 5 (c) 14 (d) 24
22)	The direction of the resultant of the forces which represented in the opposite figure is (a) $\overrightarrow{OX}$ (b) $\overrightarrow{OX'}$ (c) $\overrightarrow{Oy}$ (d) $\overrightarrow{Oy'}$ 
23)	Two forces of magnitude F_1 , F_2 kg.wt. , where $F_1 > F_2$ and the magnitude of smallest and greatest resultant of them are 3 and 12 gm.wt. respectively , then $F_1^2 - F_2^2 = \dots\dots\dots$ (a) 12 (b) 3 (c) 9 (d) 36
24)	The magnitudes of two perpendicular forces are 6 , 8 newton then the measure of the angle between the resultant and the first force is (a) $\sin^{-1} \frac{4}{3}$ (b) $\cos^{-1} \frac{4}{3}$ (c) $\tan^{-1} \frac{4}{3}$ (d) $\tan^{-1} \frac{3}{4}$

21 Answer the following questions:

[1] Two forces of magnitudes 5, 10 Newton act on a particle and the measure of the angle between them is 120° . Find the magnitude of their resultant and the measure of the angle made by the resultant with the first force.

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[2] Two forces of magnitudes $8\sqrt{3}$, 8 Newton act on a particle and the measure of the angle between them is 150° . Find the magnitude of their resultant and the measure of the angle which it makes with the first force

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[3] Two forces of magnitudes 30, 16 Newton act on a particle. If their resultant equals 26 Newton, find the measure of the angle between the two forces.

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[4] Two forces of magnitudes 12 , 15 Newton act on a particle and the (cosine) of the angle between them equals $\frac{-4}{5}$,Find the magnitude of their resultant and the measure of the angle of inclination of the resultant to the first force.

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[5] Three forces of magnitude 5, 10 and $4\sqrt{4}$ Newton act on a particle, if the measure of the angle between the first and the second forces equals 60° , find the magnitude of the maximum and the minimum resultant for the three forces.

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[6] Two forces of magnitudes 4 , F Newton act on a particle and the angle between them is 120° . If their resultant is perpendicular to the first force, find the magnitude of F

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[7] Two forces act at a point, If the magnitude of their maximum resultant equals 32 kg.wt and the magnitude of their minimum resultant equals 12 kg.wt , Find the magnitude of each of the two forces ,then find their resultant if they enclosed between them an angle of measure 60° .

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[8] Two forces of magnitude $F, F\sqrt{2}$ N act on a particle; if their resultant is perpendicular to the first force, Find the angle between the two forces then prove that the magnitude of their resultant equals F .

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[9] Two forces of magnitudes $2, F$ Newton, the angle between them is of measure 120° find F in each of the following cases:

- (i) magnitude of the resultant = F .
- (ii) the direction of the resultant is perpendicular to the second force
- (iii) the resultant bisect the angle between the angle of two forces
- (iv) the resultant inclines by 45° to the second force

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[10] Two forces of same magnitude F kg.wt enclose between them an angle of measure 120° . If the two forces are doubled and the measure of the angle between them became 60° ,then the magnitude of their resultant increases by 11 k.g.wt than the first case . Find the magnitude of F

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[11] Two forces of magnitudes 12 , F kg.wt act on a point. The first force acts in direction of east and the second force acts in direction 60° south of the west. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° south of the east.

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[12] F1 , F2 are two forces act on a particle and enclose between them an angle of measure 120° and the magnitude of their resultant is $\sqrt{19}$ N, if the angle between them becomes 60° ,then the magnitude of their resultant becomes 7 newton. Find the value of each of F1 , F2

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[13] Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. if t he direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. find the magnitude of each force.

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Lesson (2): Forces resolution into two components

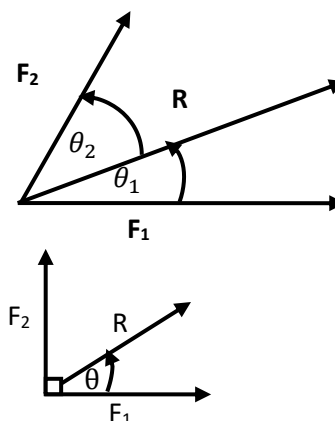
Main Points

Resolution of a force into two components:

⇒ (i) If $\vec{F}_1$, $\vec{F}_2$ are two components of a force $\vec{R}$, they make with the direction of $\vec{R}$ two angles of measures θ_1 , θ_2 respectively, then:

$$\frac{F_1}{\sin \theta_2} = \frac{F_2}{\sin \theta_1} = \frac{R}{\sin(\theta_1 + \theta_2)}$$

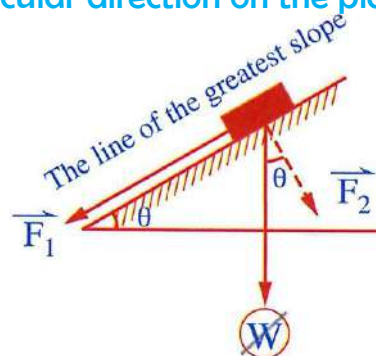
ii) If $F_1 \perp F_2 \Rightarrow F_1 = R \cos \theta$, $F_2 = R \sin \theta$



Note: 1) If $\vec{F} = (F, \theta)$, then $\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}$

2) If a body of weight (w) is placed on a smooth inclined plane with the horizontal by an angle (θ), then we can resolve the weight (w) which acts vertically downwards into two components.

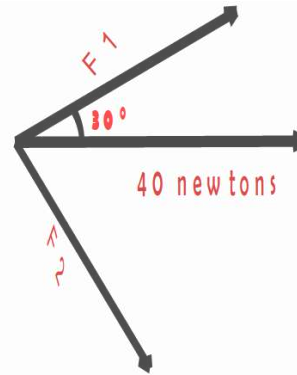
- ★ F_1 (The magnitude of the component in the direction of the line of the greatest slope)
= $w \sin \theta$
- ★ F_2 (The magnitude of the component in the perpendicular direction on the plane)
= $w \cos \theta$



1) Resolve a horizontal force 40 newtons into two perp. directions one of them inclines by 30° with the horizontal upwards

$$F_1 = 40 \cos 30 = 20\sqrt{3} \text{ N}$$

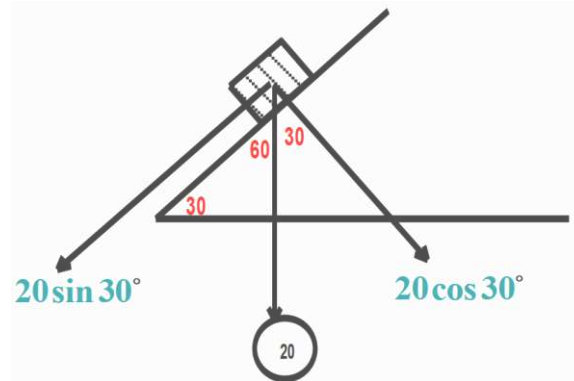
$$F_2 = 40 \sin 30 = 20 \text{ N}$$



2) A body of weight 20 newtons is placed on an inclined plane of inclination 30° to the horizontal. Find the components of the weight in the direction of the plane and the normal to it

$$F_1 = R \cos \theta = 20 \cos 30^\circ = 10\sqrt{3} \text{ newtons}$$

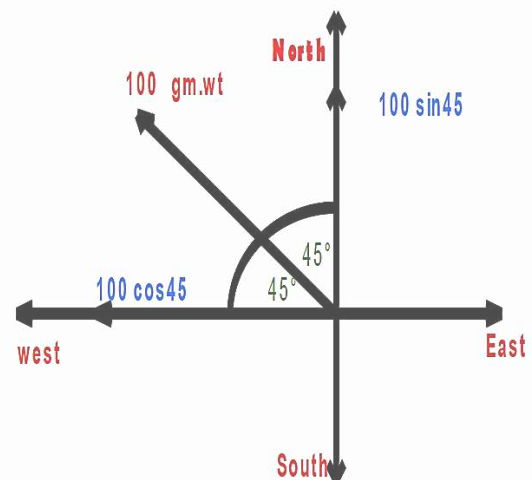
$$F_2 = R \sin \theta = 20 \sin 30^\circ = 10 \text{ newtons}$$



3) A Force of magnitude 100 gm.wt acts in the directions north west. find its components due to north and west.

$$F_1 = R \cos \theta = 100 \cos 45^\circ = 50\sqrt{2} \text{ newtons}$$

$$F_2 = R \sin \theta = 100 \sin 45^\circ = 50\sqrt{2} \text{ newtons}$$



Complete each of the following:

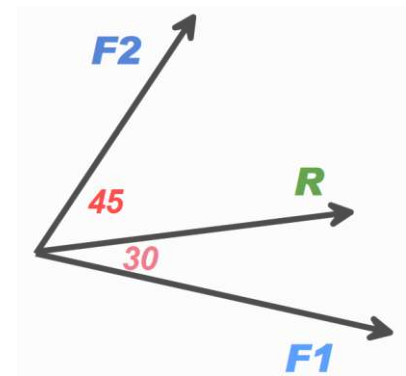
[1] A force of magnitude 6 Newton acts in direction of North. It is resolved into two perpendicular components, so its component in direction of the East equals Newton.

[2] A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components, so its component in the direction of Northern East =..... Newton.

[3] In the opposite figure:

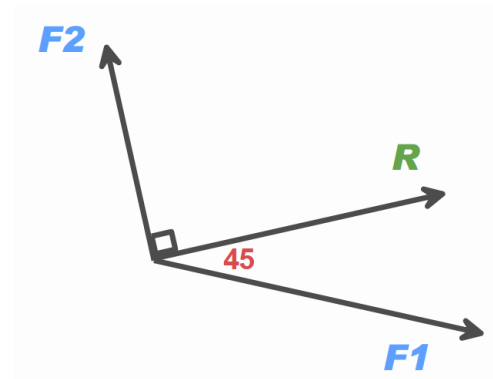
If the force $\vec{R}$ is resolved into two components $\vec{F}_1$, $\vec{F}_2$ which make with the force $\vec{R}$ two angles of measures 30° , 45° from different directions of its line of action, $||\vec{R}|| = 12$ newton,

So: $F_1 =$ Newton, $F_2 =$ Newton

[4] In the opposite figure:

If the force $\vec{R}$ is resolved into two components $\vec{F}_1$, $\vec{F}_2$ which make with the force $\vec{R}$ two angles of measure 45° , 90° from different directions of its line of action and $||\vec{R}|| = 18$ Newton,

So: $F_1 =$ Newton, $F_2 =$ Newton



[5] In the opposite figure:

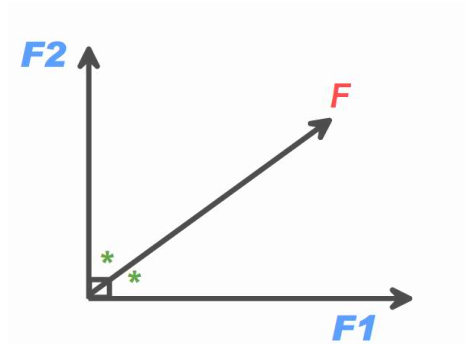
If the force $\vec{F}$ is resolved into two perpendicular components

$\vec{F}_1$, $\vec{F}_2$ and the force vector $\vec{F}$ bisects the angle between

The directions of $\vec{F}_1$, $\vec{F}_2$ and $\|\vec{F}\| = 6\sqrt{2}$ kg. wt

so: $\|\vec{F}_1\| = \dots\dots\dots$ kg wt ,

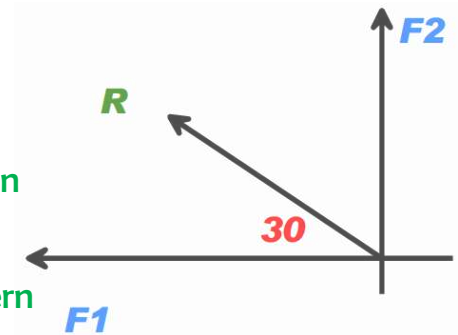
$\|\vec{F}_2\| = \dots\dots\dots$ kg wt.



[6] In the opposite figure:

Force of magnitude $12\sqrt{2}$ newton acts in direction 30° North of the west.

- Magnitude of the component of the force in the western direction = $\dots\dots\dots$ Newton.
- Magnitude of the component of the force in the northern direction = $\dots\dots\dots$ Newton.



[7] A force of magnitude 600 gm.wt acts on a particle. Find its two components in two directions making with the force two angles of measures 30° , 45° .

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[8] A force of magnitude 120 newton acts in direction of the North east. Find its two components in the direction of East and in the direction of North.

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[9] Resolve a horizontal force of magnitude 160 gm.wt in two perpendicular directions one of them inclined to the horizontal with an angle of measure 30° upwards.

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[10] A force of magnitude 18 newton acts in the direction of South. Find its two components in the 2 directions, 60° East of the South and the other direction towards 30° West of the South.

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[11] A rigid body of weight 42 newton is placed on a plane inclined to the horizontal with an angle of measure 60° . Find the two components of the weight of the body in the direction of the line of the greatest slope and the direction normal to it.

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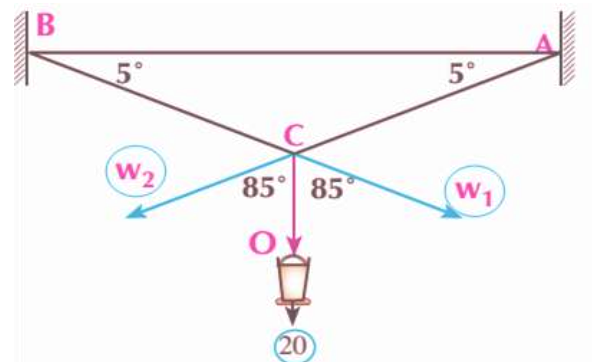
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[12] A lamp of weight 20 newton suspended by two metal rods $\overline{AC}$, $\overline{BC}$ inclined to the horizontal by two equal angles, the measure of each is 5° :
(1) Resolve the weight of the lamp into two components in the directions $\overline{AC}$, $\overline{BC}$ approximating the result to the nearest newton.



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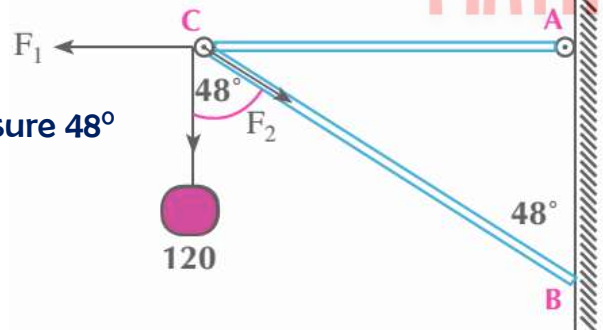
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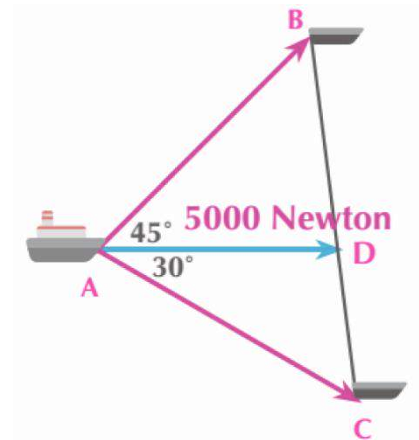
[13] In the opposite figure :

Resolve the vertical force of magnitude 120 gm.wt. into two components, one of them in the horizontal direction and the other inclined by an angle of measure 48° with the line of action of the force.



[14] In the opposite figure :

A cruiser is pulled by two ships B and C using two strands hanged to a point A on the cruiser, the measure of the angle between the two strands equals 75° , if the measure of the angle between one of the strands and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 newtons and acts on $\overrightarrow{AD}$ Find the tension in the two strands.

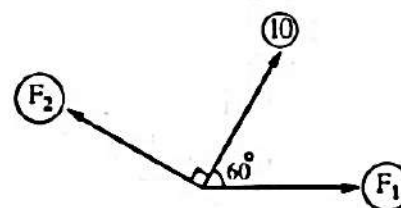


Choose the correct answer:

1)

In the opposite figure :

If the force of magnitude 10 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ inclined to the force by two angles of measures 60° and 90° respectively , then $F_2 = \dots\dots\dots$ N.



(a) $5\sqrt{3}$

(b) 10

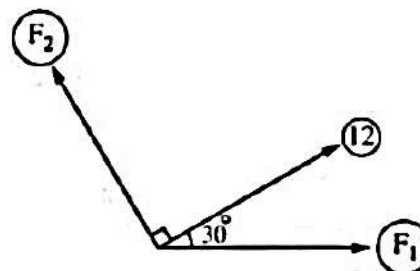
(c) $10\sqrt{3}$

(d) 20

2)

In the opposite figure :

If the force of magnitude 12 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$ inclined to the force by two angles of measures 30° and 90° respectively, then $F_2 = \dots\dots\dots$ N.



(a) 10

(b) $10\sqrt{3}$

(c) $6\sqrt{3}$

(d) $4\sqrt{3}$

3)

In the opposite figure :

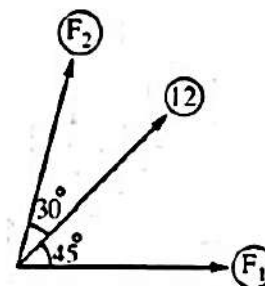
If the force of magnitude 12 N. is resolved into two components $\vec{F}_1$ and $\vec{F}_2$, then $F_1 = \dots\dots\dots$ newton.

(a) $12 \cos 75^\circ$

(b) $12 \cos 45^\circ$

(c) $6 \csc 45^\circ$

(d) $6 \csc 75^\circ$



4)

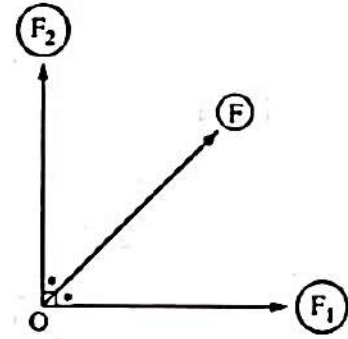
In the opposite figure :

If the force $\vec{F}$ is resolved into the two perpendicular components $\vec{F}_1$ and $\vec{F}_2$, the vector of the force $\vec{F}$ bisects the angle between the directions of $\vec{F}_1$ and $\vec{F}_2$ and $\|\vec{F}_1\| = 6\sqrt{2}$ newton, then $\|\vec{F}\| = \dots\dots\dots$ newton.

(a) 6

(b) $6\sqrt{2}$

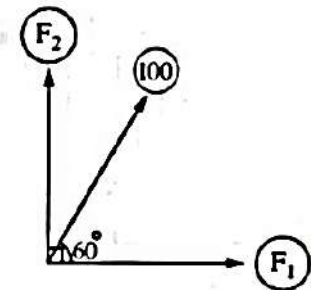
(c) 12

(d) $12\sqrt{2}$ 

5)

In the opposite figure :

If the force of magnitude 100 newton is resolved into two forces $\vec{F}_1$ and $\vec{F}_2$ and the force is measured by newton, then $(F_1, F_2) = \dots\dots\dots$

(a) $(50, 50\sqrt{3})$ (b) $(50\sqrt{3}, 10)$ (c) $(50, 50)$ (d) $(10, 10)$ 

6)

In the opposite figure :

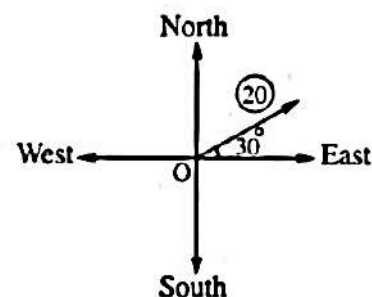
A force of magnitude 20 newton. acts in the direction 30° North of the East is resolved into two perpendicular components, then the magnitude of the component in North direction = $\dots\dots\dots$ newton.

(a) $10\sqrt{3}$

(b) 20

(c) 10

(d) 5



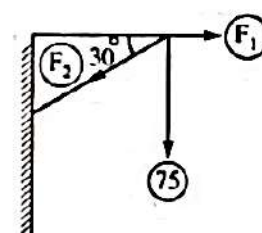
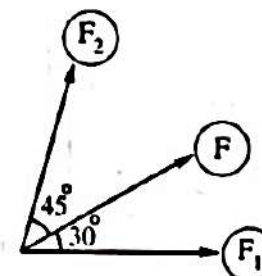
7)

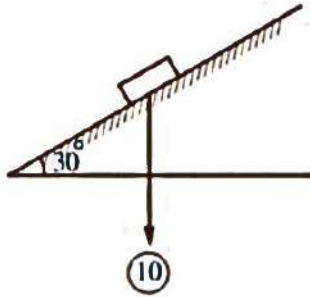
A force of magnitude $10\sqrt{2}$ gm.wt. acts in the Eastern South direction, is resolved into two perpendicular components, then the magnitude of the component in the South direction = $\dots\dots\dots$ gm.wt.

(a) 5

(b) 10

(c) $10\sqrt{2}$ (d) $5\sqrt{2}$

8)	📖 A force of magnitude 6 newton acts in direction of North. It is resolved into two perpendicular components , so its component in direction of the East of magnitude newton. (a) zero (b) 3 (c) $3\sqrt{2}$ (d) 6	
9)	📖 A force of magnitude $4\sqrt{2}$ newton acts in direction of East. It is resolved into two perpendicular components , so its component in the direction of Northern East of magnitude newton. (a) zero (b) $4\sqrt{2}$ (c) 4 (d) 6	
10)	A force of magnitude 40 newton acts vertically upwards is resolved into two components one of them is horizontal of magnitude 20 newton , then the magnitude of the other = newton. (a) 20 (b) $20\sqrt{3}$ (c) $20\sqrt{5}$ (d) $10\sqrt{3}$	
11)	In the opposite figure : A vertical force of magnitude 75 newton is resolved into two components , one of them is horizontal of magnitude F_1 and the other is of magnitude F_2 , then $F_2 =$ newton. (a) 75 (b) $75\sqrt{3}$ (c) 150 (d) $150\sqrt{3}$	
12)	In the opposite figure : The force $\vec{F}$ is the resultant of the two forces $\vec{F}_1$, $\vec{F}_2$, then $\frac{F_1 + F_2}{F} =$ (a) $\sin 30^\circ + \sin 45^\circ$ (b) $\frac{\sin 75^\circ + \sin 30^\circ}{\sin 75^\circ}$ (c) $\frac{\sin 45^\circ + \sin 30^\circ}{\sin 75^\circ}$ (d) $\frac{\sin 75^\circ}{\sin 30^\circ} + \frac{\sin 75^\circ}{\sin 45^\circ}$	

13)	In the opposite figure : If a body of weight 10 newtons is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , then the component of the weight in direction of line of the greatest slope downward = N. (a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) 5 (d) $10\sqrt{3}$	
14)	If a body of weight (W) is placed on a smooth plane inclined to horizontal by angle (θ) , so the component of its weight in direction of the plane equals (a) W (b) $W \sin \theta$ (c) $W \cos \theta$ (d) $W \tan \theta$	
15)	If a body of weight (W) is placed on an inclined smooth plane makes an angle of measure (θ) with the horizontal , then its weight component in the perpendicular direction of the plane is (a) $W \sin \theta$ (b) $W \cos \theta$ (c) $W \tan \theta$ (d) $W \csc \theta$	
16)	A body of weight (W) newton is placed on an inclined plane makes an angle of measure (θ) with the horizontal , then the components of its weight in direction line of greatest slope and its perpendicular are 7 , 24 newton respectively , then the magnitude of the weight (W) = newton. (a) 7 (b) 24 (c) 25 (d) 31	

Lesson 3: The resultant of coplanar forces meeting at a point

Main Points

- **Polygon of forces:** If a set of coplanar forces meeting at a point are represented by sides of a polygon taken in the same cyclic order, then the magnitude of the resultant of these forces equals the length of side that closed this polygon in the opposite cyclic order.
- In a perpendicular coordinate plane if a set of coplanar forces that meeting at a point at this point, and the algebraic sum of the components of these forces in the two perpendicular direction are X, Y ,

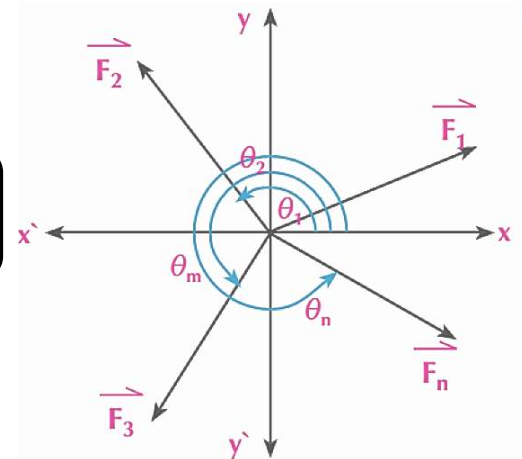
Where $X = \sum F_i \cos \theta_i$ and $Y = \sum F_i \sin \theta_i$

then:

$$R = \sqrt{X^2 + Y^2}$$

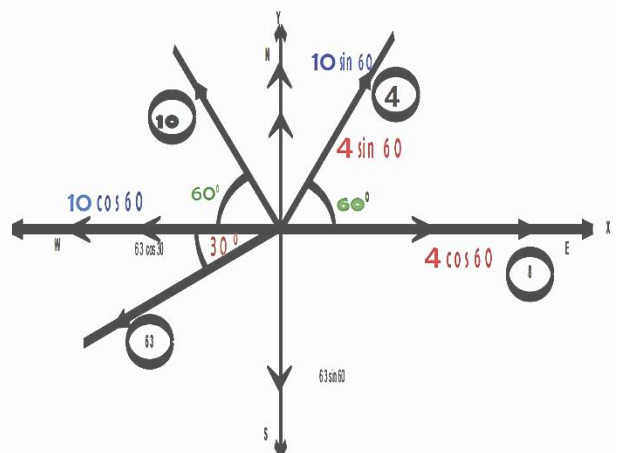
and

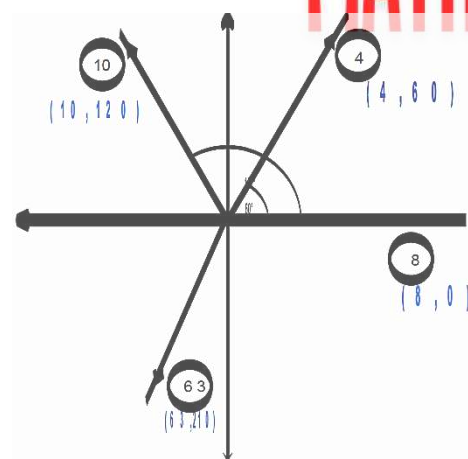
$$\tan \theta = \frac{Y}{X}$$



where θ is the polar angle of R .

1) Four coplanar forces act at a point, the 1st of magnitude 8N and acts towards east, the 2nd of magnitude 4N and acts in direction 60° north of east, the 3rd of magnitude 10N and acts in direction 60° north of west, and the 4th of magnitude $6\sqrt{3}$ and acts in direction 30° south of west, find the magnitude and direction of their resultant.





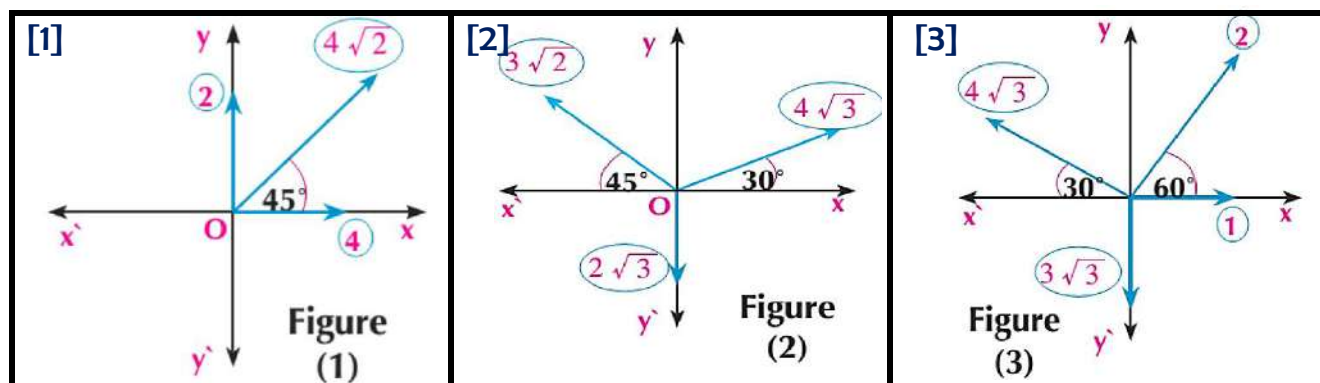
Complete each of the following:

[1] If the forces $\vec{F}_1 = 2\hat{i}$, $\vec{F}_2 = \hat{i} - 2\hat{j}$, $\vec{F}_3 = 6\hat{j}$ then: the magnitude of the resultant of the forces = and its direction

[2] If the forces $\vec{F}_1 = 2\hat{i} - 2\hat{j}$, $\vec{F}_2 = 4\hat{i} - 8\hat{j}$, $\vec{R} = 2a\hat{i} - 3b\hat{j}$ then $a = \dots\dots\dots$, $b = \dots\dots\dots$

[3] If the forces $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$, $\vec{R} = 6\hat{i} - 4\hat{j}$ then $a = \dots\dots$, $b = \dots\dots$

Find the magnitude and the direction of resultant of the forces shown in each of the following figures:



[4]

Isosceles triangle

Figure (4)

[5]

Rectangle it's dimensions are 6cm , 8cm

Figure (5)

[6]

Regular hexagon

Figure (6)

Answer the following questions:

① The forces 3 , 6 , $9\sqrt{3}$ and 12 kg.wt act on a particle and the measure of the angle between the first and the second is 60° , between the second and the third is 90° and between the third and the fourth is 150° . Find the magnitude and the direction of resultant of these forces.

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② Three forces of magnitudes 10 , 20 , 30 newton act at a particle. The first acts towards the east and the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° South of the west. Find the magnitude and the direction of resultant of these forces.

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③ Four forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 gm.wt act on a particle, the first acts in the east direction and the second acts in the direction 60° north of the east and the third acts in the direction 30° north of the west and the fourth acts in the direction making an angle of 60° South of the east. Find the magnitude and direction of resultant of these forces.

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④ ABC is an equilateral triangle, M is the point of intersection of its medians. The forces of magnitudes 15 , 20 , 25 newton act on a particle in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$. Find the magnitude and the direction of the resultant of these

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⑤ ABCD is a square of side length 12 cm , $H \in \overline{BC}$ so $BH = 5$ cm. Forces of magnitudes 2 , 13 , $4\sqrt{2}$ and 9 gm.wt act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

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⑥ If $\vec{F}_1 = 5\hat{i} + 3\hat{j}$, $\vec{F}_2 = a\hat{i} + 6\hat{j}$, $\vec{F}_3 = -14\hat{i} + b\hat{j}$ are three coplanar forces meeting at a point and their resultant $\vec{R} = (10\sqrt{2}, 135^\circ)$, then find the values of a and b.

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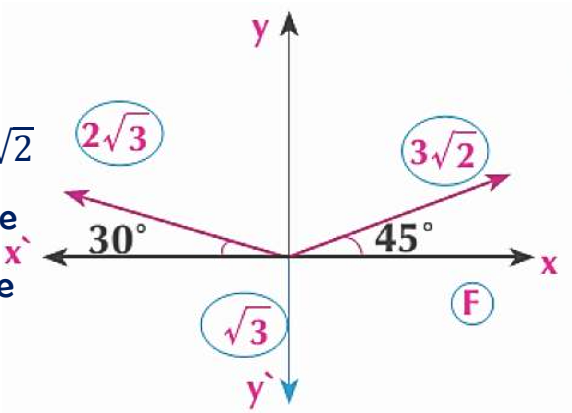
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⑦ In the opposite figure:

If the magnitude of the resultant of the forces equals $3\sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force.



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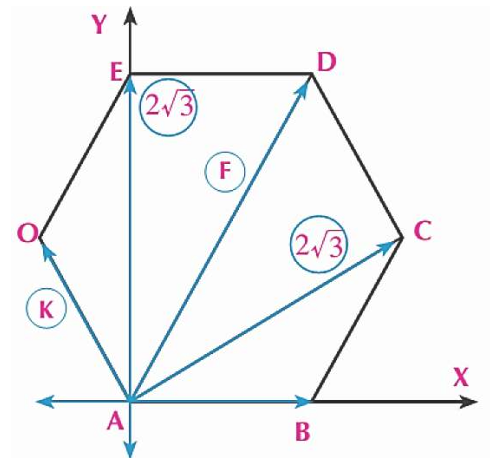
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⑧ In the opposite figure:

If the magnitude of the resultant of the forces equals 20 Kg.wt and acts in the direction of $\overrightarrow{AD}$ Find the values of F and K.



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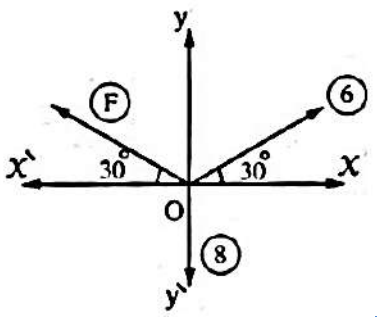
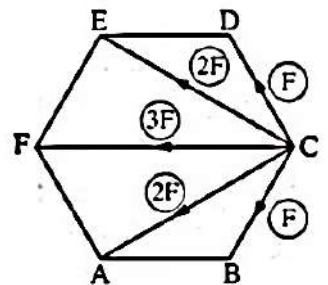
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Choose the correct answer:

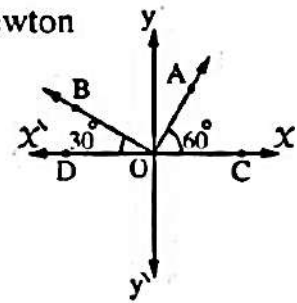
1)	If $\vec{F}_1 = \hat{i} - \hat{j}$, $\vec{F}_2 = 2\hat{i} - 4\hat{j}$, $\vec{R} = 2a\hat{i} - 3b\hat{j}$, then $a + b = \dots\dots\dots$ (a) 3 (b) $3\frac{1}{3}$ (c) $3\frac{1}{6}$ (d) 12	
2)	 If $\vec{F}_1 = 3\hat{i} - 2\hat{j}$, $\vec{F}_2 = a\hat{i} - \hat{j}$, $\vec{F}_3 = 4\hat{i} - b\hat{j}$, $\vec{R} = 6\hat{i} - 4\hat{j}$, then $(a, b) = \dots\dots\dots$ (a) (1, -1) (b) (-1, 1) (c) (-1, -1) (d) (1, 1)	
3)	If $\vec{F}_1 = 3\hat{i} + 2\hat{j}$, $\vec{F}_2 = a\hat{i} + 7\hat{j}$, $\vec{F}_3 = -12\hat{i} + b\hat{j}$ are three coplanar forces meeting at a point and the resultant $\vec{R} = (6\sqrt{2}, \frac{3}{4}\pi)$, then $a - b = \dots\dots\dots$ (a) -3 (b) 3 (c) zero (d) 6	
4)	Three coplanar forces $\vec{F}_1 = 6\hat{i} + 7\hat{j}$, $\vec{F}_2 = a\hat{i} - 9\hat{j}$, $\vec{F}_3 = 5\hat{i} + b\hat{j}$ act at a particle and they are in equilibrium , then $a + 2b = \dots\dots\dots$ (a) -9 (b) 5 (c) 7 (d) -7	
5)	If the resultant of the forces in the given figure acts in direction of y-axis , then $F = \dots\dots\dots$ force unit. (a) 2 (b) 6 (c) 8 (d) 14	
6)	The resultant of the forces in the opposite figure acts in direction of (a) $\vec{CD}$ (b) $\vec{CE}$ (c) $\vec{CF}$ (d) $\vec{CA}$	

7)

In the opposite figure :

The magnitude of four coplanar forces are 1, 2, $4\sqrt{3}$, $3\sqrt{3}$ newtonact at point O in the direction of $\overrightarrow{OX}$, $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OY}$, $m(\angle AOC) = 60^\circ$, $m(\angle BOD) = 30^\circ$,

then the magnitude and the direction of the resultant of the forces is

(a) (4, 180°)(b) (4, 0°)(c) (3, 0°)(d) (5, 90°)

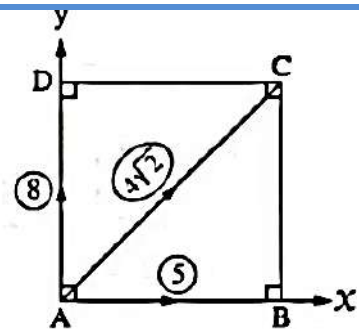
8)

In the opposite figure :

ABCD is a square, the forces of magnitudes

5, 8, $4\sqrt{2}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AC}$ respectively

, then the polar form of the resultant is

(a) (5, 54°)(b) (15, 60°)(c) (15, $53^\circ 8'$)(d) (13, 90°)

9)

In the opposite figure :

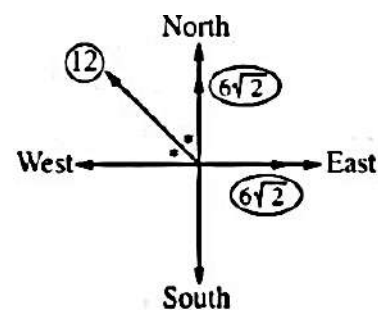
The direction of the resultant of the forces is

(a) South.

(b) East.

(c) West.

(d) North.



10)

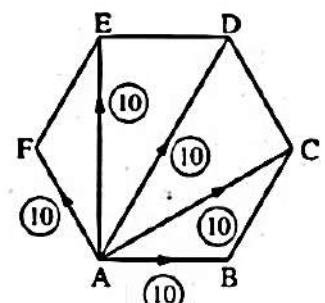
In the opposite figure :

Five equal forces each of magnitude 10 newton act

at one vertex of a regular hexagon and in direction of the other vertices of the hexagon, then the magnitude of the resultant of these forces = newton.

(a) 50

(b) 20

(c) $30\sqrt{3}$ (d) $20 + 10\sqrt{3}$ 

11)

In the opposite figure :

ABCDEF is a regular hexagon , forces of magnitudes $2, 4\sqrt{3}, 8, 2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}$ and $\overline{AF}$ respectively.

First : The magnitude of their resultant = kg.wt.

(a) $14 + 6\sqrt{3}$

(b) 20

(c) $20\sqrt{3}$

(d) $20 + \sqrt{3}$

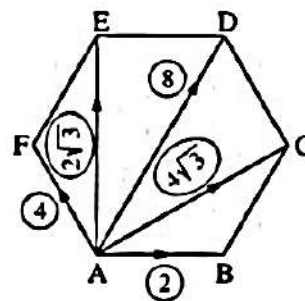
Second : The direction of the resultant inclined by an angle of measure with $\overline{AB}$

(a) 30°

(b) 45°

(c) 60°

(d) 90°



12)

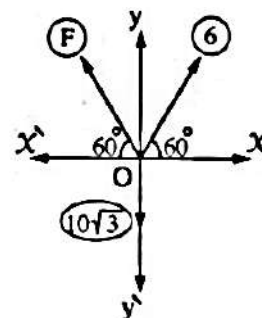
If the resultant of the forces represented in the opposite figure acts in X-axis , then $F = \dots\dots\dots$ newton.

(a) 10

(b) 14

(c) 18

(d) 6



13)

In the opposite figure :

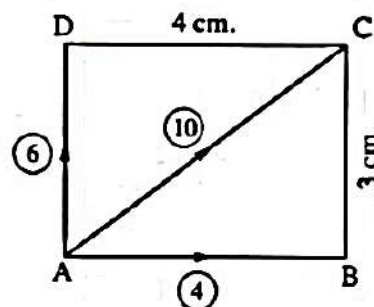
ABCD is a rectangle $AB = 4$ cm. , $BC = 3$ cm. forces 4 N , 10 , 6 N acts along $\overline{AB}, \overline{AC}, \overline{AD}$ respectively. The resultant of these forces makes with $\overline{AB}$ an angle of measure

(a) 45°

(b) 60°

(c) 30°

(d) $\sin^{-1}\left(\frac{3}{5}\right)$



Equilibrium of a rigid body under the effect of coplanar forces meeting at a point

Main Points

- If a set of coplanar forces meeting at a point are completely represented by a side of a closed polygon, taken in the same cyclic order, then the forces are **equilibrium**.
- A set of coplanar forces meeting at a point is to be in **equilibrium**, if
 - 1) The sum of the algebraic components in the direction $\overrightarrow{OX} = \text{zero}$
 - 2) The sum of the algebraic components in the direction $\overrightarrow{OY} = \text{zero}$.
- **A body is in equilibrium under the effect of only two forces, means :**
The two forces are equal in magnitude, opposite in direction and their line of action are on the same straight line.
- **Transfer the point of action of the force:** if a force acts on a rigid body ,then it's possible to transfer its point of action to another point in the body on the same line of action of the force without any change of its effect on the body.
- **The equilibrium of a body under the effect of three forces:**
If a set of coplanar forces meeting at a point are represented by the sides of triangle taken in the same cyclic order, then these forces are equilibrium

1) If the three forces $\vec{F}_1 = 2\vec{i} + 5\vec{j}$, $\vec{F}_2 = \vec{i} - 8\vec{j}$, $\vec{F}_3 = m\vec{i} + n\vec{j}$, meeting at a point and in equilibrium, find the value of "m" and "n".

The system is in equilibrium

$$\therefore \vec{R} = 0 \quad \therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$\therefore (2 + 1 + m) \vec{i} + (5 - 8 + n) \vec{j} = 0 \quad \therefore (m + 3) \vec{i} + (n - 3) \vec{j} = 0\vec{i} + 0\vec{j}$$

$$\therefore m + 3 = 0 \quad \therefore \underline{m = -3} \quad \text{and} \quad \therefore n - 3 = 0 \quad \therefore \underline{n = 3}$$

2) Three coplanar forces 8, 10, and 12 newtons act at a point, if the forces are in equilibrium, find the measure of the angle between the last two forces.

The forces are in equilibrium

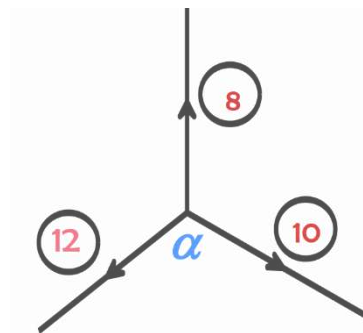
Then this resultant = 8

$$\therefore R^2 = F_1^2 + F_2^2 + 2 F_1 F_2 \cos \alpha$$

$$\therefore (8)^2 = (10)^2 + (12)^2 + 2 \times 10 \times 12 \times \cos \alpha$$

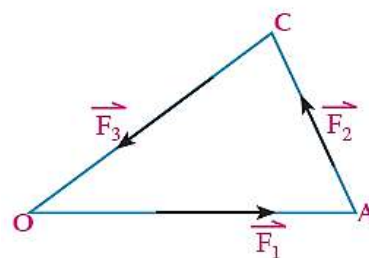
$$\therefore 240 \cos \alpha = -180 \quad \cos \alpha = -\frac{3}{4}$$

$$\therefore m(\angle \alpha) = 138^\circ 35'$$



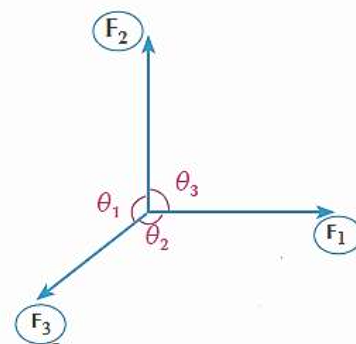
- **Triangle of forces rule:** If three forces acting at a point are in **equilibrium** and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order, then the lengths of the sides of the triangle are proportional to the magnitudes of the corresponding forces.

$$\frac{F_1}{OA} = \frac{F_2}{AC} = \frac{F_3}{CO}$$



- **Lami's rule:** If three forces meeting at a point and acting up on a particle are in **equilibrium**, then the magnitude of each force is proportional to the sine of the angle between the two other forces.

$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$



Types of questions

- (1) Body and two strings
- (2) Body, string and horizontal force
- (3) Body string and perpendicular force
- (4) Smooth inclined plane
- (5) Sphere
- (6) Rod

3) A weight 100 gm.wt is suspended by two strings of lengths 30 cm., and 40 cm., from two points on the same horizontal line with 50 cm a part. find the tension in each string.

From lami's rule

$$\therefore \frac{100}{\sin 90^\circ} = \frac{T_1}{\sin(180-\theta_1)} = \frac{T_2}{\sin(180-\theta_2)}$$

$$\therefore \frac{100}{1} = \frac{T_1}{\sin \theta_1} = \frac{T_2}{\sin \theta_2}$$

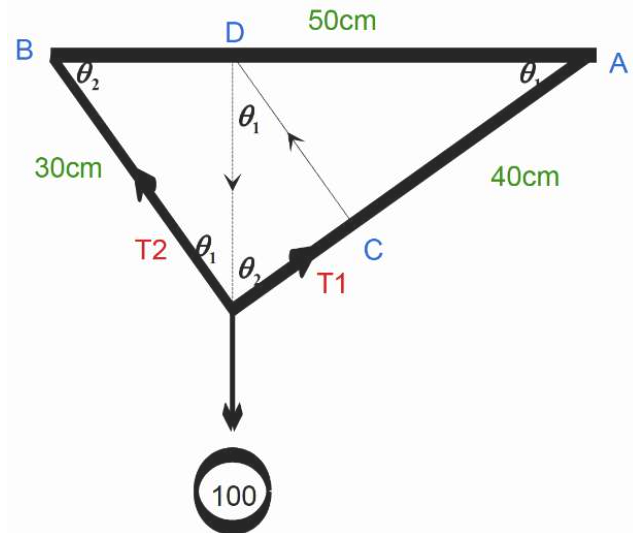
$$\sin \theta_1 = \frac{30}{50} = \frac{3}{5}$$

$$\sin \theta_2 = \frac{40}{50} = \frac{4}{5}$$

$$\therefore \frac{100}{\sin 90^\circ} = \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}}$$

$$\therefore T_1 = \frac{3}{5} \times 100 = 60 \text{ gm.wt}$$

$$\therefore T_2 = \frac{4}{5} \times 100 = 80 \text{ gm.wt}$$



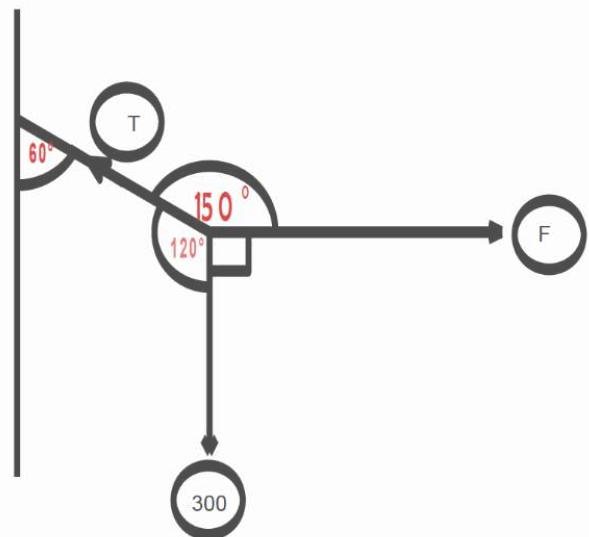
4) A body of weight 300 gm.wt is tied to an end of a string and the other end of the string is fixed at a vertical wall. If the body is pulled by a horizontal force until the string makes an angle of measure 60° with the wall in the state of equilibrium, Find the magnitude of the force and the tension in the string.

From lami's rule

$$\therefore \frac{F}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{300}{\sin 150^\circ}$$

$$\therefore T = \frac{300 \times \sin 90^\circ}{\sin 150^\circ} = 600 \text{ gm.wt}$$

$$\therefore F = \frac{300 \times \sin 120^\circ}{\sin 150^\circ} = 300\sqrt{3} \text{ gm.wt}$$



5) A body of weight 10 kg.wt is tied to an end of a string of length 170 cm., the other end of the string is fixed at a vertical wall, the body is pulled away of the wall by a horizontal force until it becomes 150 cm. apart from the wall in the state of equilibrium, Find the magnitude of F and T

In $\triangle OAB$ is right at A

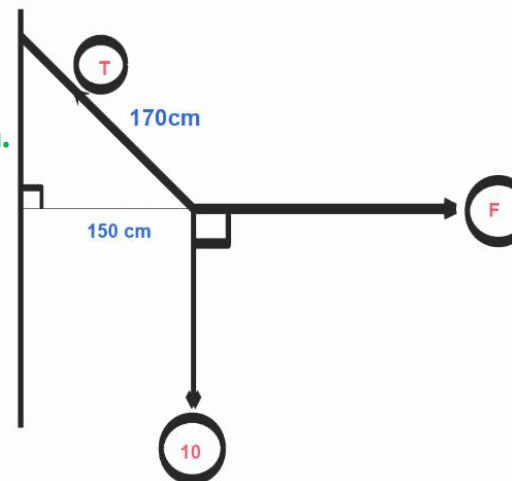
$$\therefore (OA)^2 = (OB)^2 - (AB)^2 = (170)^2 - (150)^2 \quad \therefore OA = 80 \text{ cm.}$$

$\triangle OAB$ is the triangle of forces

$$\therefore \frac{10}{80} = \frac{T}{170} = \frac{F}{150}$$

$$\therefore T = \frac{10 \times 170}{80} = \frac{85}{4} \text{ gm.wt}$$

$$\therefore F = \frac{10 \times 150}{80} = \frac{75}{4} \text{ gm.wt}$$



6) A body of mass 6 kg.wt is placed on smooth inclined plane 30° with the horizontal, is supported by force, Find this force and the reaction of the plane in each of the following cases.

first: the force is horizontal

second: the force is inclined by 30° to the plane

first: the force is horizontal

By using Lami's rule:

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 90^\circ} = \frac{6}{\sin 120^\circ}$$

$$\therefore F = \frac{6 \times \sin 150^\circ}{\sin 120^\circ} = 2\sqrt{3} \text{ gm.wt}$$

$$\therefore r = \frac{6 \times \sin 90^\circ}{\sin 120^\circ} = 4\sqrt{3} \text{ gm.wt}$$

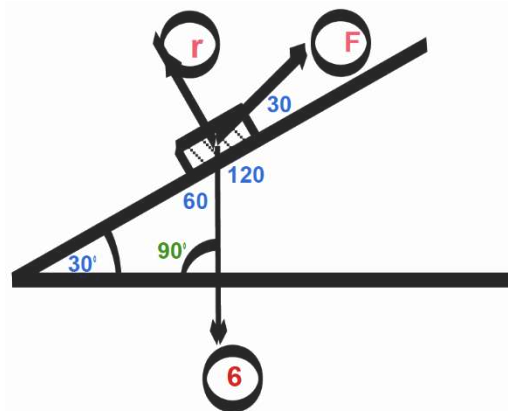
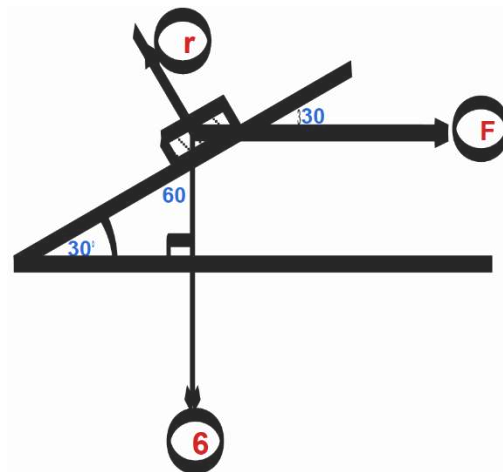
second: the force is inclined by 30° to the plane

By using Lami's rule:

$$\therefore \frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{6}{\sin 60^\circ}$$

$$\therefore F = \frac{6 \times \sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3} \text{ gm.wt}$$

$$\therefore r = \frac{6 \times \sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3} \text{ gm.wt.}$$



8) A body of weight "w" newtons is suspended by two strings, the first inclines by angle of measure θ to the vertical passing over a smooth pulley and carries at its other end a body of weight 12 newtons, the other string form with the vertical 30° , passing over a smooth pulley and carries at its other end a body of weight 8 newtons. Find θ and F

By using Lami's rule:

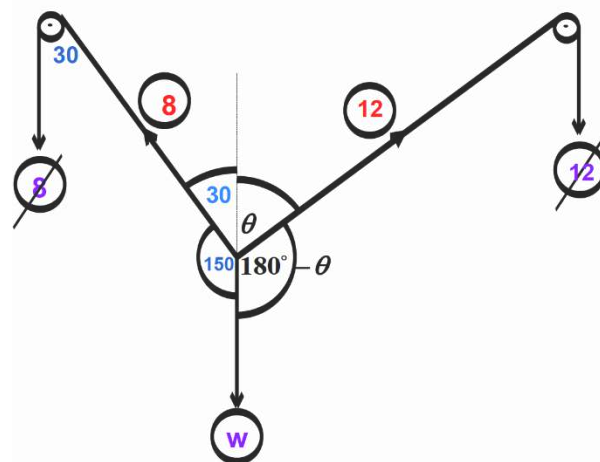
$$\therefore \frac{W}{\sin(30^\circ + \theta)} = \frac{12}{\sin 150^\circ} = \frac{8}{\sin(180^\circ + \theta)}$$

$$\therefore \sin \theta = \frac{8 \times \sin 150^\circ}{12} = \frac{1}{3}$$

$$\therefore m(\angle \theta) = 19^\circ 28'$$

$$\therefore \frac{W}{\sin(30^\circ + 19^\circ 28')} = \frac{12}{\sin 150^\circ}$$

$$\therefore W = \frac{12 \times \sin 49^\circ 28'}{\sin 150^\circ} \approx 18 \text{ newtons}$$



9) A body weight 400 gm.wt, is suspended by a string at a point A from a point B on the string, another string is attached and pulled horizontally by second string BC passing over a smooth fixed pulley and carries at its other end a body 300 gm.wt, find the inclination of AB with vertical and the tension in each of the two strings AB , BC

By using triangle of forces:

$\triangle ADB$ is the triangle of forces

$$\therefore \frac{T_1}{BD} = \frac{T_2}{AB} = \frac{W}{AD}$$

$$\therefore \frac{300}{BD} = \frac{T_2}{AB} = \frac{400}{AD} \quad \therefore \frac{300}{BD} = \frac{400}{AD}$$

$$\therefore \frac{300}{400} = \frac{BD}{AD} = \frac{3}{4} \quad \therefore \tan A = \frac{3}{4}$$

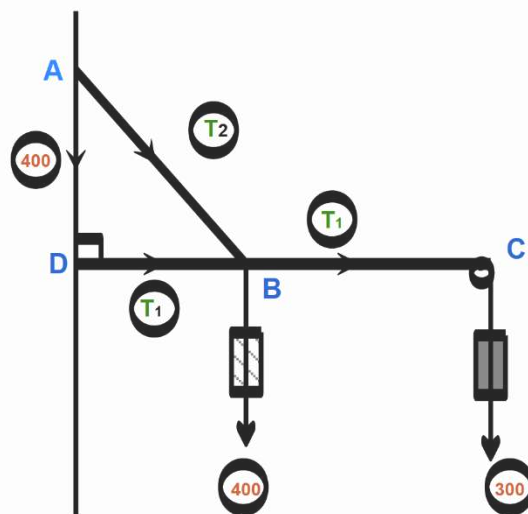
$$\therefore m(\angle \theta) = 36^\circ 52'$$

$$\therefore \text{angle } \theta \text{ inclination} = 36^\circ 52'$$

By using Lami's rule

$$\therefore \frac{300}{\sin 36^\circ 52'} = \frac{T_2}{\sin 90^\circ}$$

$$\therefore T_2 = \frac{300 \times \sin 90^\circ}{\sin 36^\circ 52'} = 500 \text{ gm.wt.}$$



Choose the correct answer:

1)	If three forces meeting at a point and acting up on a particle are in equilibrium , then the magnitude of each force is proportional to the of the included angle between the other two forces. (a) cosine (b) sine (c) tangent (d) cotangent	
2)	If a body is kept in equilibrium under action of several forces , then the least number of forces could cause equilibrium equals (a) 1 (b) 2 (c) 3 (d) 4	
3)	If $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are three forces meeting at a point and they are in equilibrium , then the magnitude of the resultant of $\vec{F}_1$ and $\vec{F}_2$ = (a) F_1 (b) $F_1 + F_2$ (c) F_3 (d) zero	
4)	Three equal forces in magnitude meeting at a point and they are in equilibrium , then the measure of the angle between each two forces = (a) 60° (b) 90° (c) 120° (d) 150°	
5)	If $\vec{F}$ is in equilibrium with two perpendicular forces of magnitudes 8 newton and 15 newton , then F = newton. (a) 7 (b) 17 (c) 23 (d) $7\sqrt{2}$	
6)	If a force of magnitude (F) is in equilibrium with two forces of magnitudes 5 and 3 newton and the measure of the angle between them is 60° , then F = newton. (a) $\sqrt{19}$ (b) $\sqrt{34}$ (c) 7 (d) 6	
7)	The force which is in equilibrium with two perpendicular forces F , F newton makes with one of the two forces an angle of measure $^\circ$ (a) 90 (b) 120 (c) 135 (d) 150	

8)

Which of the following sets of forces could be in equilibrium ?

- ① 8 newton , 8 newton , 8 newton.
 ② 8 newton , 8 newton , 16 newton.
 ③ 8 newton , 8 newton , 20 newton.
 (a) ① only. (b) ② only. (c) ① , ② (d) ② , ③

9)

Three coplanar forces are in equilibrium act at a particle , the measure of the angle between the first two forces is 60° , and between the second and third forces is 150° then the ratio between forces is

- (a) $1 : 1 : \sqrt{3}$ (b) $1 : 2 : \sqrt{3}$ (c) $\sqrt{2} : \sqrt{3} : 1$ (d) $\sqrt{3} : \sqrt{3} : 1$

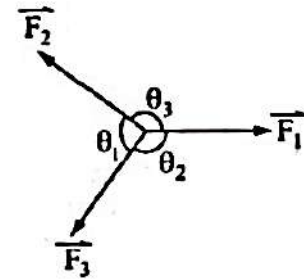
10)

If a body is in equilibrium under action of three forces as shown in the figure.

Which of the following statements is true ?

- ① $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \text{zero}$ ② $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{\text{zero}}$
 ③ $\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$

- (a) ① , ② only. (b) ② , ③ only.
 (c) ③ only. (d) ① , ② , ③

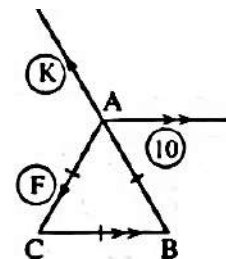


11)

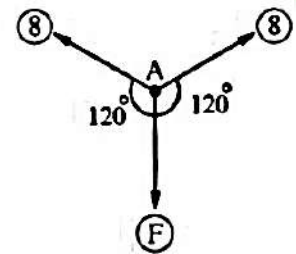
In the opposite figure :

If the three coplanar forces meeting at a point and in equilibrium , then $F = \dots\dots\dots$ N.

- (a) 10 (b) 12 (c) 20 (d) 13



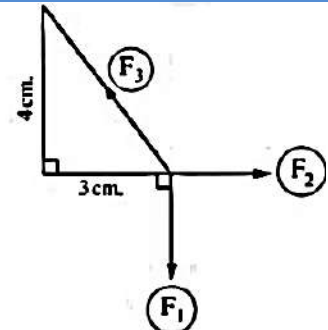
- 12) Particle A is balanced under action of three forces as shown in the opposite figure where $\vec{F}$ is balanced with two forces the magnitude of each is 8 newton and makes an angle of measure 120° with each of them, then $F = \dots\dots\dots$ newton.



- (a) zero (b) 8
(c) 16 (d) $8 \sin 120^\circ$

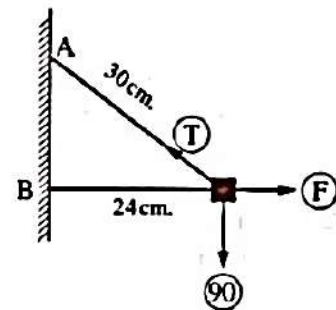
- 13) In the opposite figure :

A body is in equilibrium under action of three forces meeting at a point of magnitudes F_1 , F_2 and F_3 newton and the sides of the right-angled triangle are parallel to the lines of action of the forces in the same cyclic order, then $F_1 : F_2 : F_3 = \dots\dots\dots$



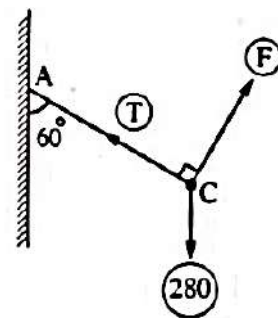
- (a) 3 : 4 : 5 (b) 3 : 5 : 4 (c) 4 : 5 : 3 (d) 4 : 3 : 5

- 14) In the opposite figure :
A body of weight 90 gm.wt. is attached to the end of a string of 30 cm. long. The body is pulled by a horizontal force. It comes to equilibrium when it is 24 cm. apart from the wall $\overline{AB}$ then $T - F = \dots\dots\dots$ gm.wt.



- (a) 150 (b) 120
(c) 50 (d) 30

- 15) In the opposite figure :
A lamp of weight 280 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertical by an angle of measure 60° , then $\frac{F}{T} = \dots\dots\dots$



- (a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

16)

In the opposite figure :

The weight of a body is 20 kg.wt , the body is in

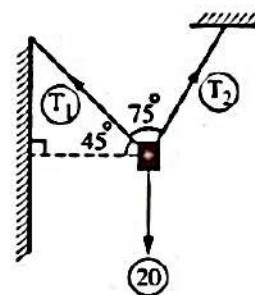
equilibrium then $\frac{T_1}{T_2} = \dots\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{2}{3}$

(d) $\frac{\sqrt{3}}{2}$



17)

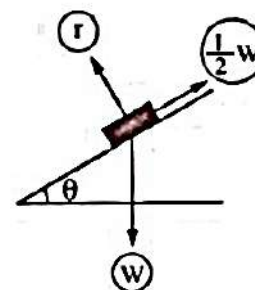
In the opposite figure :If the body is in equilibrium under action of forces shown , then $m(\angle \theta) = \dots\dots\dots$

(a) 30°

(b) 60°

(c) 45°

(d) 15°



18)

In the opposite figure :

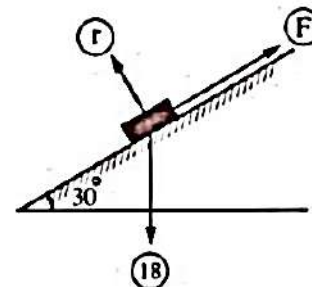
A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° , it is kept in equilibrium by a force of magnitude F newton in the direction of the plane upward , then $F + R = \dots\dots\dots$ newton.

(a) $6\sqrt{3}$

(b) $9\sqrt{3}$

(c) $18\sqrt{3}$

(d) $9 + 9\sqrt{3}$



19)

The weight of a body is 6 kg.wt. It is placed on a smooth inclined plane makes an angle 30° to the horizontal and kept in equilibrium by a horizontal force , then the magnitude of this horizontal force = $\dots\dots\dots$ kg.wt.

(a) $\sqrt{3}$

(b) $2\sqrt{3}$

(c) $4\sqrt{3}$

(d) 6

20)

In the opposite figure :

A body of weight (W) is hanged by two strings.

The two strings inclined to the horizontal as shown

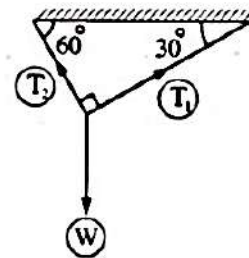
in the figure , then $T_1 = \dots\dots\dots$

(a) $\frac{1}{3} W$

(b) $\frac{1}{2} W$

(c) $\frac{\sqrt{3}}{3} W$

(d) $\frac{\sqrt{3}}{2} W$



21)

In the opposite figure :

A body of weight 150 gm.wt. is in equilibrium by suspending it by two perpendicular strings of lengths 60 cm. and 45 cm. , and the other two ends C and B are on a horizontal line , then :

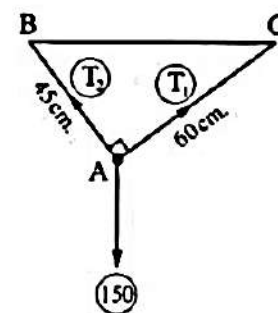
 $T_2 - T_1 = \dots\dots\dots$ gm.wt.

(a) 120

(b) 90

(c) 60

(d) 30



22)

A body of weight 28 kg.wt. is suspended by two perpendicular strings , if the measure of the angle between one strings and the line of the weight is 120° , then the magnitude of the tension of this strings equals $\dots\dots\dots$ kg.wt.

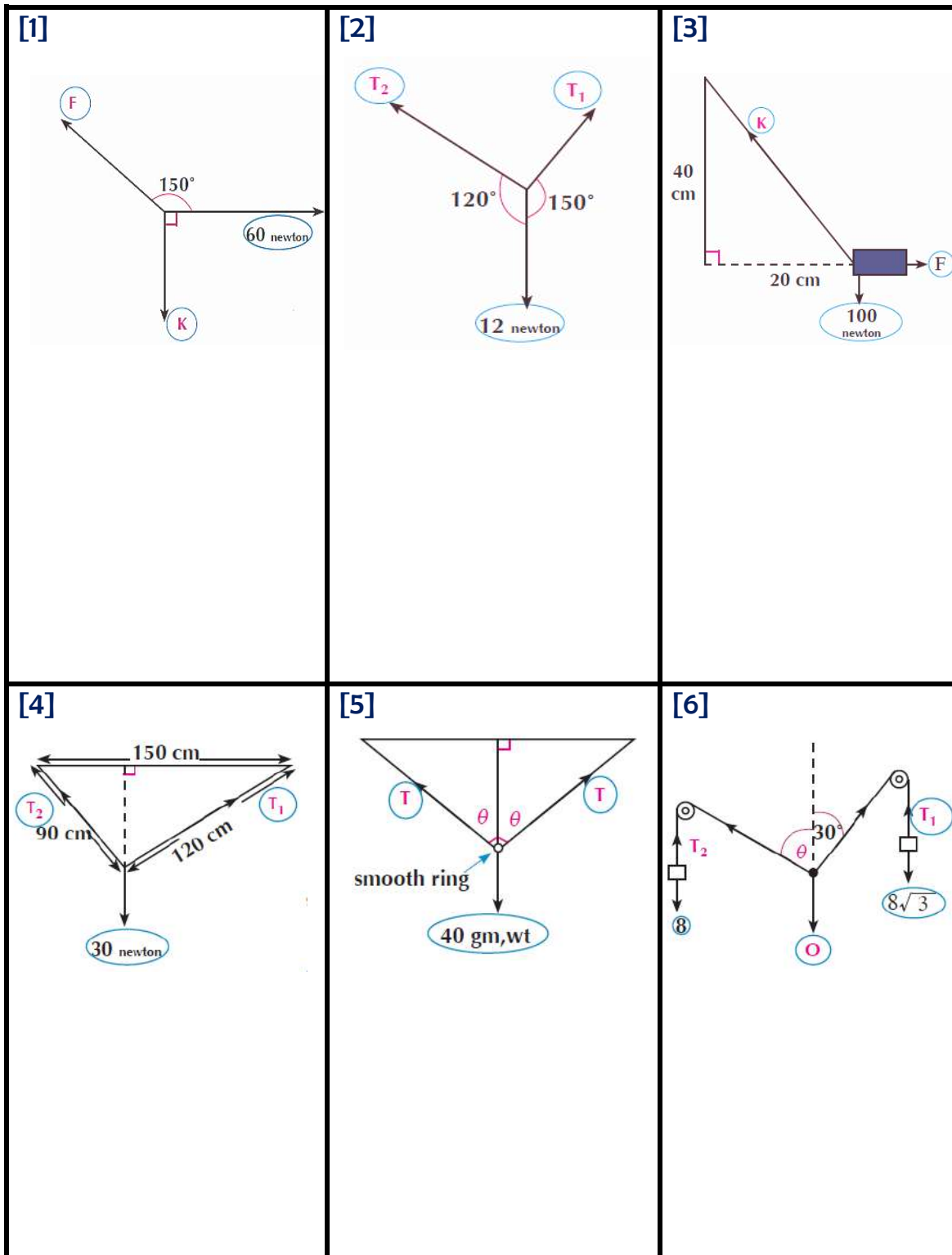
(a) 14

(b) 28

(c) $14\sqrt{3}$

(d) $28\sqrt{3}$

Each figure from the following figures represents a set of coplanar equilibrium force meeting at a point. Find the value of the unknown (Force or angle):



Answer the following questions

[1] A weight of magnitude 60 gm.wt is suspended from one end of a string of length 28 cm. The other end is fixed at a point in the ceiling of the room, A force acts on the body so that the body becomes in equilibrium when it is about 14 cm vertically down the ceiling, If the force is in equilibrium position when it is normal to the string. Find the magnitude of each of the force and the tension in the string.

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[2] A weight of magnitude 200 gm.wt is hanged (suspended) by two strings of lengths 60 cm, 80 cm from two points on one horizontal line. The distance between them is 100 cm. Find the magnitude of the tension in each of the two strings.

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[3] A particle of weight 200 gm.wt is hanged (suspended) by two light strings.

One of the them inclines to the vertical with an angle of measure θ and the other string inclines to the vertical with an angle of measure 30° . If the magnitude of the tension in the first string equals 100 gm.wt, find θ and magnitude of the tension of the second string.

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[4] A body of weight 800 gm.wt is placed on a smooth plane inclined to the horizontal by angle of measure θ so that $\sin \theta = 0.6$. The body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

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[5] A body of weight (W) newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and the body is kept in equilibrium by the effect of force of magnitude 36 newton acts in direction of the line of the greatest slope upwards. Find the magnitude of the weight of the body and the magnitude of the reaction of the plane.

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Follow: The Equilibrium

1) From a point on the surface of a homogenous sphere, a light string is tied whose other end is attached to a point in a vertical smooth wall. if the sphere is stable on the wall, find the tension in the string and the reaction of the wall if the weight of the sphere is W newtons and acts at its center and the string inclined to the vertical by 30°

In this problem we find that the sphere is in equilibrium state under the action of three forces:

(1) The weight (W) act at M

(2) The reaction of the wall (R)

the wall is smooth, then the reaction is perpendicular to the plane, therefore, it passes through the center of the sphere M

(3) The tension in the string (T)

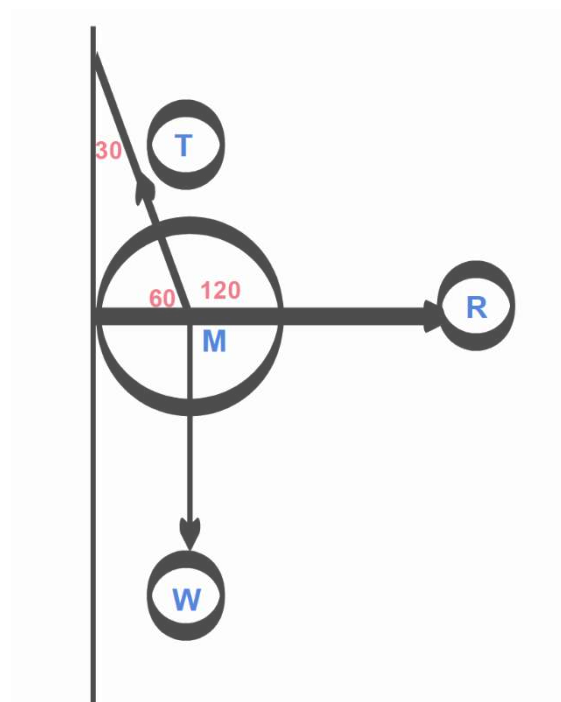
Since the line of the action of the weight and the reaction of the wall passes through the center M , then the line of action of the third force (the tension) must be pass through the point M

; By using Lami's Rule:

$$\therefore \frac{W}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin 150^\circ}$$

$$\therefore T = \frac{W \times \sin 90^\circ}{\sin 120^\circ} = \frac{2\sqrt{3}}{3} W \text{ Newtons}$$

$$\therefore R = \frac{W \times \sin 150^\circ}{\sin 120^\circ} = \frac{\sqrt{3}}{3} W \text{ Newtons}$$



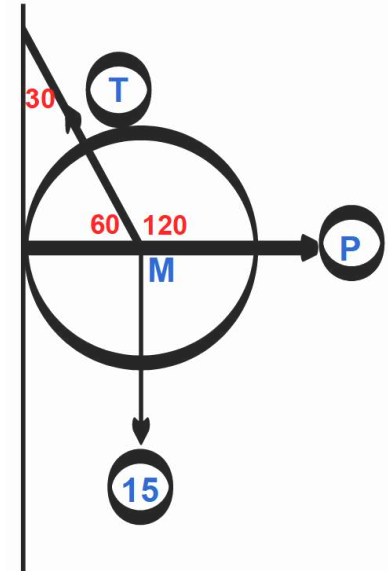
2) A smooth sphere of weight 15 newtons is on a smooth vertical wall and suspended by a light string from a point on its surface the other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere, if the length of the string equals the radius of the sphere, find the pressure on the wall and the tension in the string

By using Lami's Rule:

$$\therefore \frac{15}{\sin 120^\circ} = \frac{P}{\sin 150^\circ} = \frac{T}{\sin 90^\circ}$$

$$\therefore P = \frac{15 \times \sin 150^\circ}{\sin 120^\circ} = 5\sqrt{3} \text{ Newtons}$$

$$\therefore T = \frac{15 \times \sin 90^\circ}{\sin 120^\circ} = 10\sqrt{3} \text{ Newtons}$$



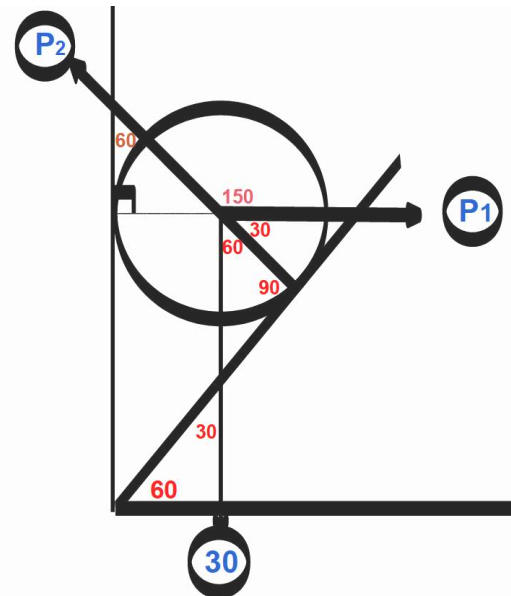
3) A smooth iron sphere of weight 30 newtons rests between a vertical smooth wall and an inclined smooth plane inclined by 60° with the horizontal, find the pressure on each of the wall and the plane

By using Lami's Rule:

$$\therefore \frac{P_1}{\sin 120^\circ} = \frac{P_2}{\sin 90^\circ} = \frac{30}{\sin 150^\circ}$$

$$\therefore P_1 = \frac{30 \times \sin 120^\circ}{\sin 150^\circ} = 30\sqrt{3} \text{ Newtons}$$

$$\therefore P_2 = \frac{30 \times \sin 90^\circ}{\sin 150^\circ} = 60 \text{ Newtons}$$

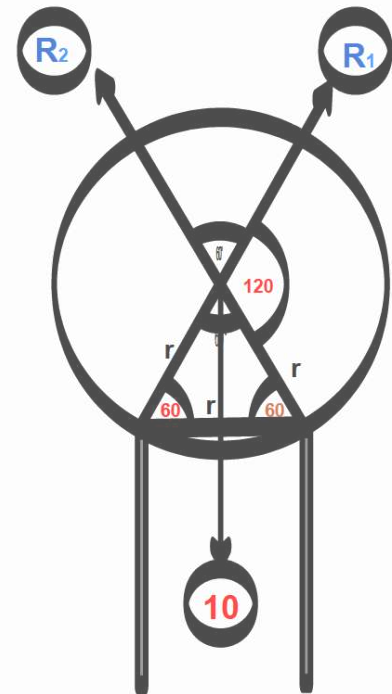


4) A sphere rests between two parallel rods lie in the same horizontal plane, the distance between them equals the radius of the sphere find the pressure on each rod given that the weight of the sphere is 10 newtons.

By using Lami's Rule:

$$\therefore \frac{R_1}{\sin 150^\circ} = \frac{R_2}{\sin 150^\circ} = \frac{10}{\sin 90^\circ}$$

$$\therefore R_1 = R_2 = \frac{10 \times \sin 150^\circ}{\sin 60^\circ} = \frac{10\sqrt{3}}{3} \text{ Newtons}$$



5) A uniform rod of 1 metre long and 30 newtons weight is suspended from both its ends by two strings their other ends are fixed at one point in the ceiling. if the two strings are perp. one of them is 60 cm., find the tension in each string when it is suspended freely and in equilibrium.

By using triangle of forces:

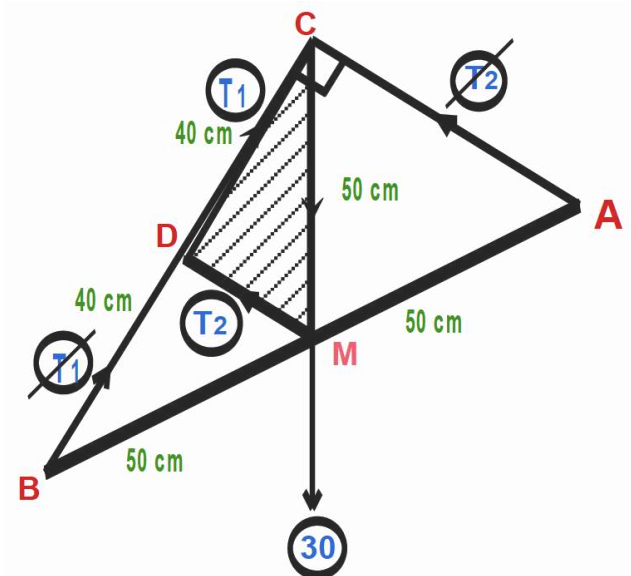
ΔDMC is the triangle of forces

$$\therefore \frac{30}{CM} = \frac{T_1}{DC} = \frac{T_2}{MD}$$

$$\therefore \frac{30}{50} = \frac{T_1}{40} = \frac{T_2}{30}$$

$$\therefore T_1 = \frac{40 \times 30}{50} = 24 \text{ Newtons}$$

$$\therefore T_2 = \frac{30 \times 30}{50} = 18 \text{ Newtons}$$



6) AB is a uniform rod of length 120 cm., and weight 300 gm.wt its end A is attached to a hinge fixed at a vertical wall, the other end B is attached to a string BC of length 150 cm., this string is fixed at a point C on the wall and lies vertically above A, the rod is in equilibrium when it is in horizontal position, find the tension in the string and the reaction of the hinge at A.

ΔABC is right at A $\therefore AC = \sqrt{150^2 - 120^2} = 90 \text{ cm.}$

$\therefore D$ is midpoint of AB, E is midpoint of BC

$\therefore AE$ is a median drawn from the vertex of right angle

$$\therefore AE = \frac{1}{2} BC = 75 \text{ cm}$$

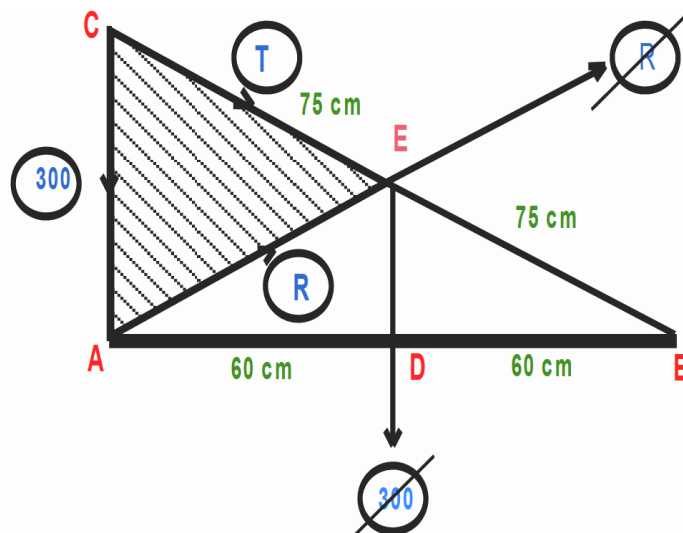
By using triangle of forces:

ΔAEC is the triangle of forces

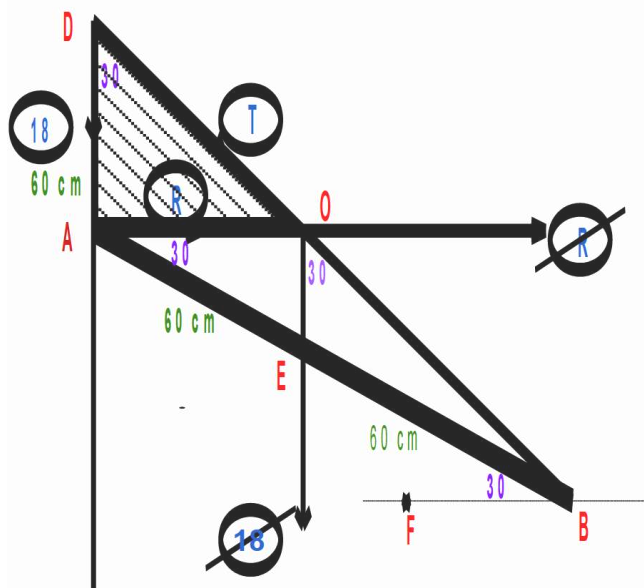
$$\therefore \frac{300}{CA} = \frac{R}{AE} = \frac{T}{CE}$$

$$\therefore \frac{300}{90} = \frac{R}{75} = \frac{T}{75}$$

$$\therefore R = T = \frac{75 \times 300}{90} = 250 \text{ gm.wt}$$

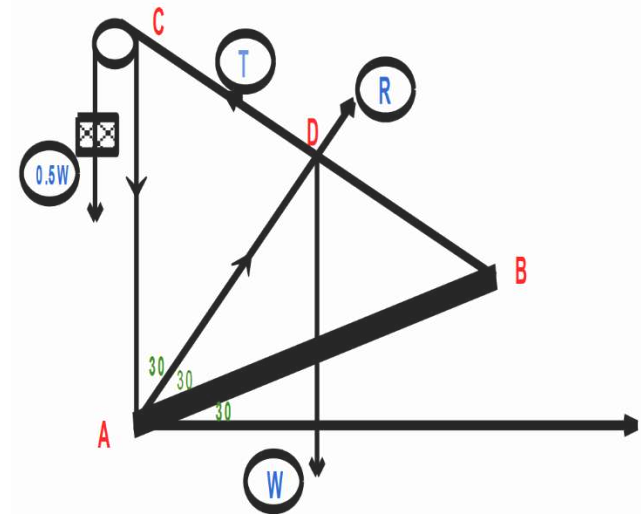


7) AB is a uniform rod of length 120 cm., and weight 18 km.wt it rotates about a hinge at its end A and its end B is pulled by a light string whose other ends is tied to a point D above A, the rod in equilibrium when it inclined to the horizontal at 30° , if the reaction of the hinge is horizontal. Find the tension of the string and the reaction of the rod in the wall



8) AB is a uniform rod hinged at A, the other end B is tied to a string pass over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the angle of inclination of the rod to the horizontal in state of equilibrium given that

$$AC = AB$$



EXERCISES

[1] A smooth sphere of Wight 200 gm.wt and of radius length 30 cm. rest against a smooth vertical wall. It is suspended at a point of its surface by means of a string of length 20 cm. ,and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. find the tension in the string and the reaction of the wall.

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[2] A smooth sphere of Wight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertically an angle of measure 30° . find the tension in the string and the reaction of the wall.

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[3] A smooth metal sphere of weight 3 Newton at rest (stable) between a smooth vertical wall and a smooth plane inclined to the vertical wall with angle of measure 30° . Find the pressure on each of the vertical wall and the inclined plane.

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[4] A rod of length 50 cm and weight 20 newton was hanged (suspended) from its terminals with two strings such that the two ends are fixed in one point. If the length of the two strings are 30 cm , 40 cm respectively. Find the tension in each of the two strings.

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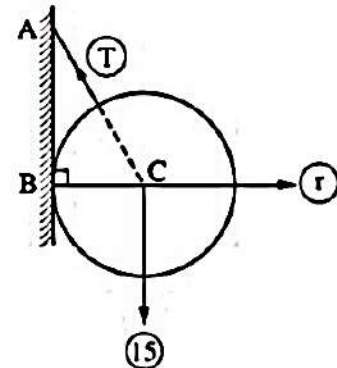
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Choose the correct answer:

1)

In the opposite figure :

A solid uniform sphere of weight 15 gm.wt. and radius length 10 cm. is in equilibrium by a string of length 10 cm. attached to a point of its surface and the other end of the string is fixed at a point in the vertical smooth plane above the tangency point , then $(r, T) = \dots\dots\dots$



(a) $(4\sqrt{3}, 8\sqrt{3})$

(b) $(5\sqrt{3}, 10\sqrt{3})$

(c) $(5, 10)$

(d) $(5\sqrt{3}, 8\sqrt{3})$

2)

In the opposite figure :

If the sphere is in equilibrium

, then $T - r = \dots\dots\dots$ newton

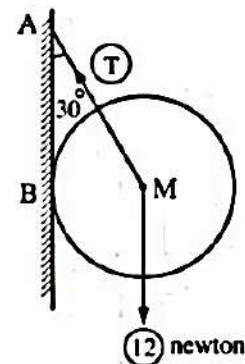
(Where r is the magnitude of the wall reaction on the sphere)

(a) $8\sqrt{3}$

(b) $4\sqrt{3}$

(c) 4

(d) 8



3)

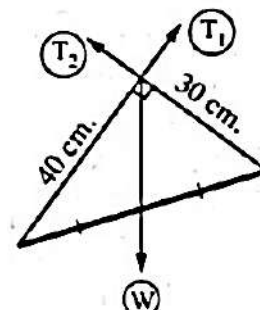
In the opposite figure : $T_1 : T_2 : W = \dots\dots\dots$

(a) 5 : 3 : 4

(b) 8 : 5 : 4

(c) 4 : 3 : 5

(d) 5 : 4 : 3



4)

In the opposite figure :

$\overline{AB}$ is a uniform rod with length 40 cm. and weight 30 newton is connected to a hinge at A. If the rod kept in equilibrium horizontally by a light string connected to the rod at B and C where C is located on the wall just above A , $AC = 40$ cm.

First : The reaction of the hinge $r = \dots\dots\dots$ newton.

(a) 30

(b) 20

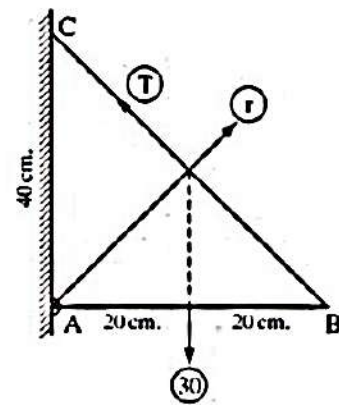
(c) $40\sqrt{2}$ (d) $15\sqrt{2}$

Second : The tension of the string $T = \dots\dots\dots$ newton.

(a) $15\sqrt{2}$

(b) 30

(c) 20

(d) $40\sqrt{2}$ 

Unit Two

LESSON 1:

Friction — equilibrium of a body placed
on a horizontal rough plane

LESSON 2:

Equilibrium of a body on a rough
inclined plane

Lesson (1): Friction — equilibrium of a body placed on a horizontal rough plane

Smooth surface and rough surface

Smooth surface: It is a hypothetical surface which has no friction force at all.

Rough surface: It is a surface which has a friction force appears when the body tends to move.

The static friction's force

It is a hidden force appears only as trying to move a body on a rough surface.

The force of limiting static friction

It is the friction's force when the magnitude of the friction force is at its limiting value (maximum value) at which the body is about to move. It is denoted by $\vec{F}_s$.

At this value the motion is just about to begin and the friction is then a limiting static friction it is denoted by $\vec{F}_s$, then the two equations of equilibrium are $P = F_s$, $R = W$

The coefficient of static friction

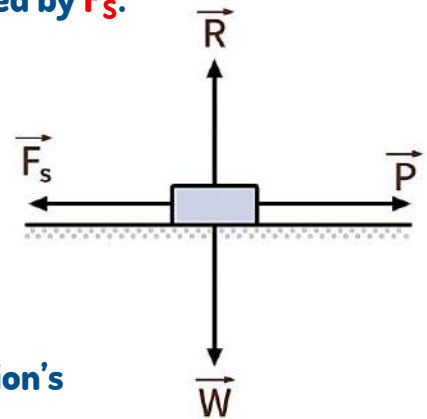
The ratio between the two magnitudes of the limiting static friction's force ($\vec{F}_s$) and the normal reaction ($\vec{R}$) is called the coefficient of friction between the two surfaces in contact and it is denoted by μ_s ,

i.e. $\mu_s = \frac{F_s}{R}$, then $F_s = \mu_s R$

Kinetic friction force

If a body moves on a rough surface; it is subjected to the kinetic friction ($\vec{F}_k$) which acts in opposite direction to the motion and $F_k = \mu_k R$

Where μ_k , is the kinetic friction coefficient and R is the normal reaction.



The resultant reaction

The resultant reaction is denoted by $\vec{R}$

It is the resultant of the normal reaction $\vec{R}$

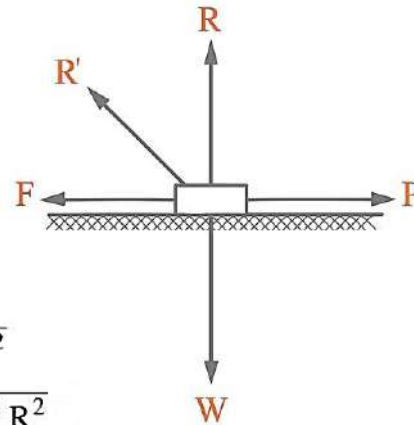
and the force of friction $\vec{F}$

i.e. $\vec{R} = \sqrt{R^2 + F^2}$

In the case of limiting friction we find that $\vec{R} = \sqrt{R^2 + F_s^2}$

$\therefore F_s = \mu_s R$ (the limiting friction) $\therefore \vec{R} = \sqrt{R^2 + \mu_s^2 R^2}$

$\therefore \vec{R} = R \sqrt{1 + \mu_s^2}$



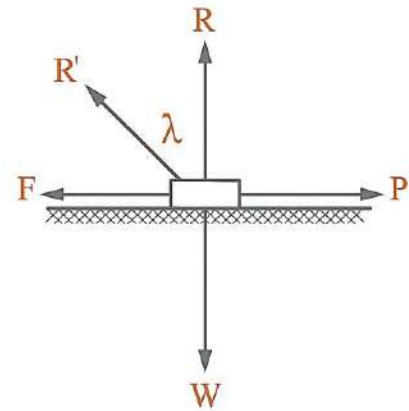
The angle of friction

It is the angle included between the resultant reaction and the normal reaction when the magnitude of the friction's force is the limiting value

i.e. when $F_s = \mu_s R$

The measure of angle of friction is denoted by the symbol (λ)

, then $\tan \lambda = \frac{F_s}{R}$ but $\frac{F_s}{R} = \mu_s \therefore \mu_s = \tan \lambda$



i.e. The tangent of the angle of friction equals the coefficient of static friction.

$\therefore \vec{R} = R \sqrt{1 + \mu_s^2} \therefore \vec{R} = R \sqrt{1 + \tan^2 \lambda} = R \sqrt{\sec^2 \lambda}$

$\therefore \vec{R} = R \sec \lambda$

- Notice : $\vec{R}$, $\vec{F}_s$ are components of $\vec{R}$ in direction of plane and the perpendicular to it in case of limiting friction i.e. $F_s = \vec{R} \sin \lambda$, $R = \vec{R} \cos \lambda$

The equilibrium of a body on a horizontal rough plane

If a body of weight (W) is placed on a horizontal rough plane and a force of magnitude (P) inclines to the horizontal upwards at an angle of measure θ acted on it, then the body in the equilibrium position is in equilibrium under the action of three forces which are:

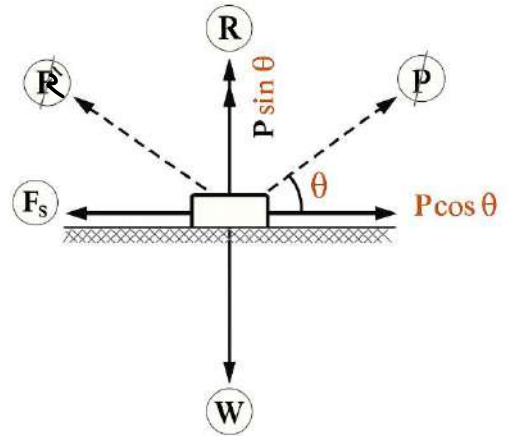
- Resolving the force P into two components;

one in the horizontal direction and the other

on the perpendicular direction to it,

these components are of magnitudes $P \cos \theta$ and $P \sin \theta$

, then the equations of equilibrium of the body are:



$$F_s = P \cos \theta \quad (1)$$

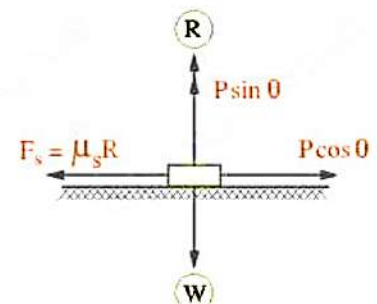
$$, R + P \sin \theta = W \quad (2)$$

- If the body is about to move , then the friction is limiting friction

i.e. $F_s = \mu_s R$, then the equations of equilibrium are :

$$\mu_s R = P \cos \theta \quad (1)$$

$$, R + P \sin \theta = W \quad (2)$$



Examples

1) A body of weight 15 kg.wt. is placed on a rough horizontal plane. A horizontal force of magnitude 5 kg.wt. acted on the body, then the body became about to move:

1. Find the coefficient of static friction between the body and the plane.
2. If weights of magnitude 3 kg.wt. is put on the body. Find the magnitude of the horizontal force which acts on the body and what it carries of weights to be about to move.

Sol.

1. $\therefore$ The body is about to move

$\therefore$ The friction force is limiting and its magnitude $= \mu_s R_1$

$\therefore$ The two equations of equilibrium are :

$$\mu_s R_1 = 5 \quad (1) , \quad R_1 = 15 \quad (2)$$

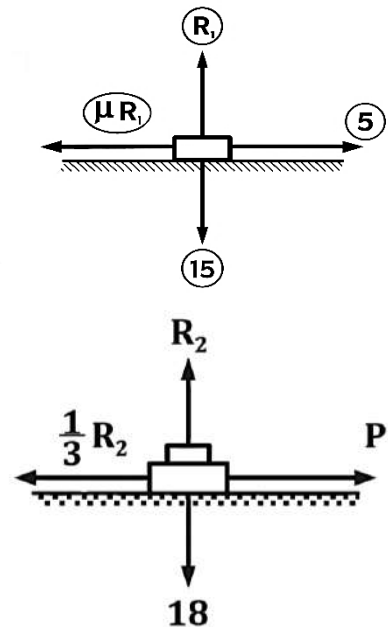
$$\text{Substituting from (2) in (1) : } \therefore \mu_s \times 15 = 5 \quad \therefore \mu_s = \frac{1}{3}$$

2. The magnitude of the weight of the body and what it carries $= 15 + 3 = 18$ kg.wt.

$\therefore$ The two equations of equilibrium are :

$$P = \frac{1}{3} R_2 \quad (1) , \quad R_2 = 18 \quad (2)$$

$$\text{Substituting from (2) in (1) : } \therefore P = \frac{1}{3} \times 18 = 6 \text{ kg.wt.}$$



2) A body of weight 15 kg.wt. is placed on a rough horizontal plane.

If the measure of the angle of friction between the body and the plane is 30° . Find:

1. The horizontal force which is enough to make the body about to move.
2. The force inclined to the plane at an angle of measure 30° upwards and makes the body about to move on the plane also.

Sol.

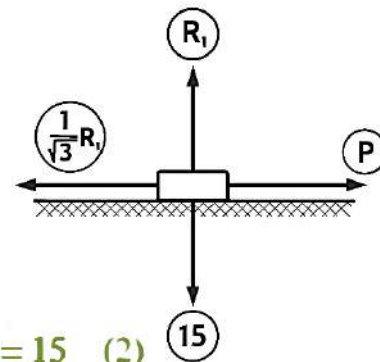
1. If the force is horizontal :

$\therefore$ The body is about to move on the plane.

$$\mu_s \text{ (the coefficient of static friction)} = \tan \lambda = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\therefore \text{The two equations of equilibrium are : } P = \frac{1}{\sqrt{3}} R_1 \quad (1) \quad , \quad R_1 = 15 \quad (2)$$

$$\text{Substituting from (2) in (1) : } \therefore P = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ kg.wt.}$$

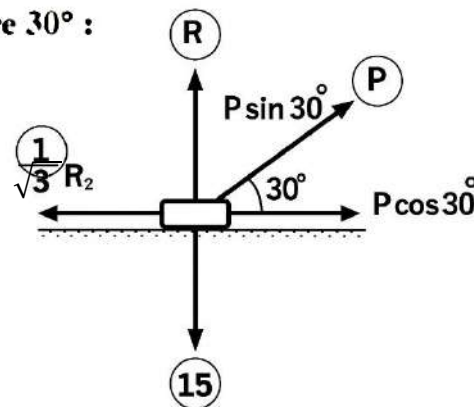


2. If the force inclines to the horizontal at an angle of measure 30° :

$\therefore$ The two equations of equilibrium are :

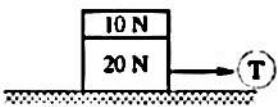
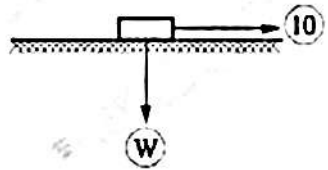
$$P \cos 30^\circ = \frac{1}{\sqrt{3}} R_2 \quad \therefore R_2 = \frac{3}{2} P \quad (1)$$

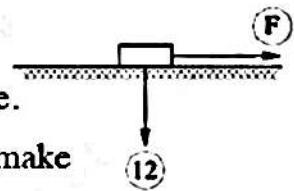
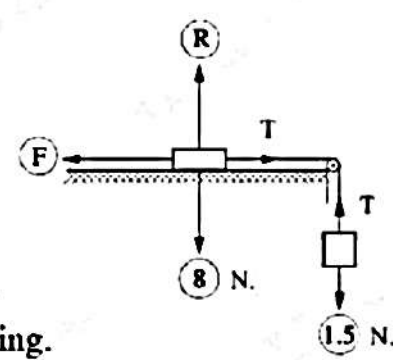
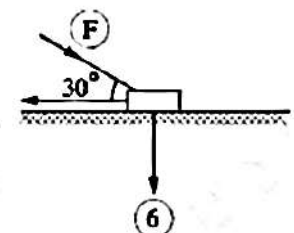
$$R_2 + P \sin 30^\circ = 15 \quad \therefore R_2 + \frac{1}{2} P = 15 \quad (2)$$



$$\text{Substituting from (1) in (2) : } \therefore \frac{3}{2} P + \frac{1}{2} P = 15 \quad \therefore P = 7.5 \text{ kg.wt.}$$

60

6)	If θ is the measure of the angle between the normal reaction and the resultant reaction when the friction is limiting and 2θ is the measure of the angle between the resultant reaction and the limiting static friction , then the coefficient of static friction = (a) $\frac{\sqrt{3}}{3}$ (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{3}$
7)	Wael pushes a box of books to his car on a rough horizontal plane. The weight of the box and the books is 80 N. , and the coefficient of static friction between the box and the plane is 0.25 , then the horizontal force that Wael can use to make the box about to move equals N (a) 20 (b) 60 (c) 80 (d) 320
8)	A body of weight (w) kg.wt. is placed on a rough horizontal plane and the coefficient of static friction between the body and the plane = $\frac{2}{5}$ If a horizontal force of magnitude 45 kg.wt. acts on the body and make it about to move , then the weight of the body = kg.wt. (a) 22.5 (b) 90 (c) 112.5 (d) 225
9)	A body consists of two parts their weights are 20 N. , 10 N. , are placed as shown in the figure , a horizontal force T acts on it and makes the body about to move , if the upper body of weight 10 N. is removed and the tension is still acting as it was , then the body  (a) moves in the tension direction. (b) stays about to move. (c) stays at rest and not about to move. (d) moves in the opposite direction of tension.
10)	In the opposite figure : A body of weight (w) kg.wt. is placed on a rough horizontal surface. A horizontal force of magnitude 10 kg.wt. acts on the body and makes the body about to move. The resultant reaction of the surface is $10\sqrt{2}$ kg.wt. , then the weight of the body (w) = kg.wt.  (a) 10 (b) 20 (c) $10\sqrt{2}$ (d) $20\sqrt{2}$

11)	If λ is the measure of the angle between force of limiting friction and resultant reaction , then μ (the coefficient of static friction) = (a) $\tan \lambda$ (b) $\sin \lambda$ (c) $\cos \lambda$ (d) $\cot \lambda$	
12)	If the limiting friction force is 60 newton and coefficient of friction is 0.75 , then magnitude of resultant reaction equals (a) 60 (b) 80 (c) 100 (d) 200	
13)	In the opposite figure : A body of weight 12 kg.wt. is placed on a rough horizontal plane. A horizontal force $\vec{F}$ (measured by kg.wt.) acted on the body to make it about to move. If the measure of the angle between the limiting static friction force and the resultant reaction is θ , where $\tan \theta = \frac{3}{4}$, then the resultant reaction $\vec{R} = \dots\dots\dots$ kg.wt. (a) 9 (b) 20 (c) 15 (d) 12	
14)	In the opposite figure : The coefficient of friction equals $\frac{1}{4}$ and the body is equilibrium on the plane , then (a) the friction force = 2 newton. (b) the resultant reaction is perpendicular to the plane. (c) the friction between the body and the plane is limiting. (d) the friction between the body and the plane is not limiting.	
15)	In the opposite figure : A body of weight 6 N. is placed on a rough horizontal plane. A force of magnitude 6 N. acts on the body and acts in direction inclined with the horizontal at an angle of measure 30° The body becomes about to move , then the measure of the angle between the resultant reaction ($\vec{R}$) and the force (F) equals (a) 30° (b) 60° (c) 120° (d) 150°	

16)

A body of weight (w) gm.wt. is placed on a rough horizontal plane. Two horizontal forces of magnitude 6 , 10 gm.wt. act on the body and makes the body about to move. If the measure of the included angle between the two forces is 60° , then the coefficient of static friction between the body and the plane is $\frac{1}{3}$, then $w = \dots\dots\dots$ gm.wt.

(a) 18

(b) 21

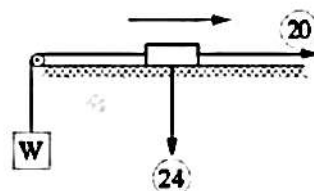
(c) 30

(d) 42

17)

In the opposite figure :

A body of weight 24 newton is placed on a horizontal rough table , the coefficient of the static friction between the body and the table = $\frac{1}{2}$, the body is tied to the end of a light inelastic string passes over a small smooth pulley at the edge of the table and the other end is tied to another body of weight (W) newton suspended vertically. A horizontal force of magnitude 20 newton acts on the body to make it about to move in the direction of the force , then $W = \dots\dots\dots$ newton.



(a) 24

(b) 8

(c) 6

(d) 12

Answer the following questions:

1) A boy pushes a stone of weight 56 newtons by a horizontal force of magnitude 42 newtons on a ridge, the stone was about to move. Find the coefficient of static friction between the stone and the ridge.

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2) A body of weight 45 kg.wt. is placed on a horizontal rough plane, the coefficient of static friction between the body and the plane is $\frac{1}{3}\sqrt{3}$ Find

(1) The magnitude of the horizontal force that makes the body about to move.

(2) The magnitude and the direction of the resultant reaction.

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3) A 240 kg. weight is placed on a horizontal rough plane and is pulled by a string which makes an angle of measure 30° with horizontal. If the coefficient of static friction is $\frac{\sqrt{3}}{3}$ Calculate the tension that makes the motion about to begin.

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Lesson (2): Equilibrium of a body on a rough inclined plane

If a body of weight (W) is placed on a rough plane inclined to the horizontal at an angle of measure θ and the body became in equilibrium, then it is in equilibrium under the action of two forces, they are:

(1) The force of the weight of the body W which acts vertically downwards.

(2) The force of the resultant reaction R^1 which acts

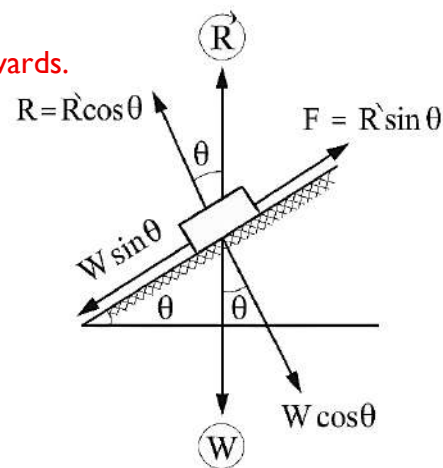
in the opposite direction of the weight, then $R^1 = W$

(3) The force of friction F which acts in a direction parallel to the

plane upwards where $F = R^1 \sin \theta$

(4) The force of the normal reaction R which acts in a

perpendicular direction to the plane upwards where $R = R^1 \cos \theta$



, then the two equations of equilibrium of the body are:

$$R = W \cos \theta, F = W \sin \theta$$

Remark (1)

If the body is about to slide under effect of its weight only,

i.e. It is about to move downwards of the plane, then

the friction is limiting and its magnitude $= \mu_s R$

, then the two equations of equilibrium are :

$$R = W \cos \theta \quad (1), \mu_s R = W \sin \theta \quad (2)$$

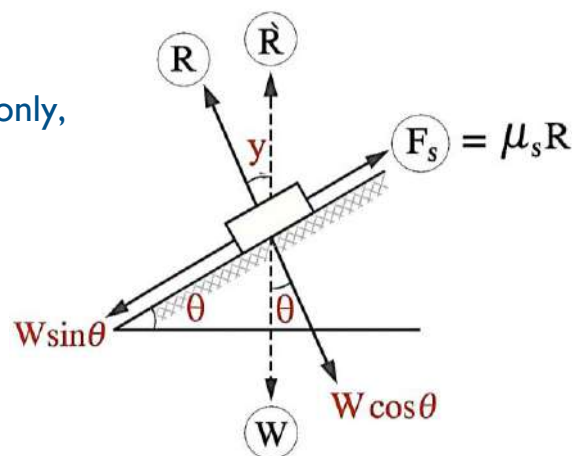
dividing (2) by (1) : We find that : $\mu_s = \frac{\sin \theta}{\cos \theta}$

$$\therefore \mu_s = \tan \theta$$

, $\because \mu_s = \tan \lambda$ where λ is the measure of the angle of friction

$$\therefore \tan \lambda = \tan \theta$$

$$\therefore \lambda = \theta$$



For example:

If a body is placed on a rough inclined plane and it was about to slide under the action of its weight only when the measure of the inclination angle is 60° , then the coefficient of static friction between the body and the plane, $\mu_s = \tan 60^\circ = \sqrt{3}$

Remark (2)

When a body of weight (W) is placed on a rough inclined plane makes angle of measure (θ) to the horizontal and (λ) is the measure of the angle of friction, then to determine if the body will be in equilibrium or it will move

1. If $\theta < \lambda$: then the body stays at rest on the plane
(i.e. It is in equilibrium and not about to move)
2. If $\theta = \lambda$: then the body is about to move downward
(i.e. It is in equilibrium and about to move downward)
3. If $\theta > \lambda$: the body can not preserve its equilibrium so it will move down the plane.

Example

A body is placed on a rough inclined plane makes an angle of measure 30° with the horizontal and the coefficient of static friction between the body and the plane is $\frac{\sqrt{3}}{5}$

State, with reason, that this body cannot preserve its equilibrium on the plane.

Sol.

$$\therefore \tan \theta = \tan 30^\circ = \frac{\sqrt{3}}{3},$$

$$\therefore \frac{\sqrt{3}}{3} > \frac{\sqrt{3}}{5}$$

$\therefore$ The body can not preserve its equilibrium on the plane.

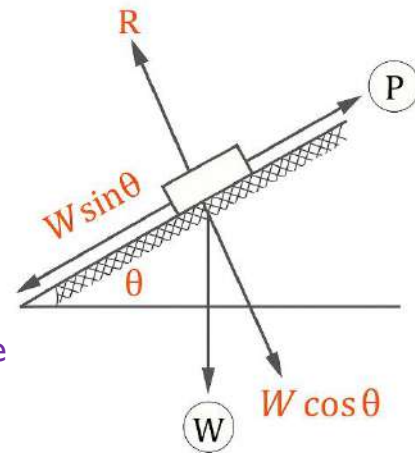
$$\therefore \mu = \tan \lambda = \frac{\sqrt{3}}{5}$$

$$\therefore \theta > \lambda$$

Remark (3)

If a force P in the direction of the line of greatest slope of the plane up, acts on a body and the body are in equilibrium, then to determine the magnitude and direction of the friction, compare between P and $W \sin \theta$

1. If $P > W \sin \theta$, then the body tends to move up
 .. The friction acts down, $P = F + W \sin \theta$
2. If $P < W \sin \theta$, then the body tends to move down
 .. The friction acts up, $P + F = W \sin \theta$
3. If $P = W \sin \theta$, then the body stays at rest on the plane and the friction (F) vanished.

**Example (1)**

A body of weight 40 newton rests on a rough plane inclined to the horizontal with an angle whose tangent $= \frac{3}{4}$, If the least force acting in the direction of the plane upwards to keep the body in equilibrium equals 16 newton
 Find the coefficient of static friction between the body and the plane.

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Example (2)

A body of weight 3 newton is placed on a plane inclined at 30° to the horizontal and the coefficient of static friction between the weight and the plane is $\frac{2}{3}$, a 2 newton force is acting on the body upwards along the line of the greatest slope. Given that the body is at rest, find the force of friction and investigate whether or not the motion is about to begin.

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Example (3)

A body of weight 30 newton is placed on a rough inclined plane. When the plane is inclined at 30° to the horizontal the body is about to slide down. If the measure of the inclination angle of the plane to the horizontal is increased to be 60° . Calculate the magnitude of:

- (1) The least force act on the body and is parallel to the line of greatest slope to prevent the body sliding.
- (2) The force act on the body and is parallel to the line of greatest slope to make the body about to move upwards.

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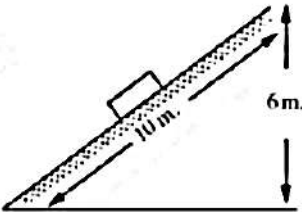
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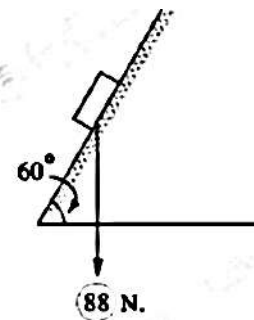
Choose the correct answer:

1)	A body of weight 6 N. was placed on a rough inclined plane so it was about to slide if the limiting friction was $3\sqrt{3}$ N. , then the measure of the inclination angle of the plane to the horizontal equals° (a) 30 (b) 45 (c) 60 (d) 90	
2)	A body is placed on a rough inclined plane and the angle of friction between the body and the plane is λ , the plane makes an angle of measure θ , then the body is in equilibrium if and only if (a) $\theta > \lambda$ (b) $\theta \geq \lambda$ (c) $\theta \leq \lambda$ (d) $\theta = 2\lambda$	
3)	A body is placed on a rough inclined plane it makes an angle of measure θ to the horizontal. The coefficient of friction between the body and the plane is $\frac{\sqrt{3}}{3}$ if the body is in equilibrium on the plane , then (a) $\theta = 30^\circ$ (b) $\theta > 30^\circ$ (c) $\theta \leq 30^\circ$ (d) $30^\circ \leq \theta < 90^\circ$	
4)	If the coefficient of static friction between a body and an inclined rough plane equals $\sqrt{3}$, then the measure of the inclination angle of the plane to the horizontal when the body is about to slide under effect of its weight only = (a) 30° (b) 45° (c) 60° (d) 75°	
5)	A body is placed on a rough inclined plane makes an angle of measure $\sin^{-1} \frac{5}{13}$ and it was about to slide under effect of its weight only , then the coefficient of static friction between the body and the plane equals (a) $\frac{5}{13}$ (b) $\frac{5}{12}$ (c) $\frac{12}{13}$ (d) $\frac{12}{5}$	
6)	In the opposite figure : The body is about to slide downwards , then the coefficient of static friction = (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$	

- 7) A body is placed on a rough plane inclined to horizontal at an angle equals the measure of the friction angle , then the body
- (a) is at rest on the plane but not about to move.
 (b) moves on the plane.
 (c) is about to move downward the plane.
 (d) is about to move upwards the plane.

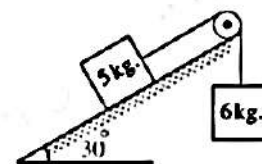
- 8) A body is placed on an inclined rough plane makes an angle of measure 60° and the coefficient of static friction between the body and the plane is $\frac{\sqrt{3}}{6}$, then the body
- (a) is about to move up the plane. (b) is about to move down the plane.
 (c) moves on the plane. (d) remains at rest but not about to move.

- 9) In the opposite figure :
 A body of weight 88 newtons placed on a rough inclined plane makes with the horizontal an angle of measure 60° . If the body about to slide , then the magnitude of the limiting static friction = newtons

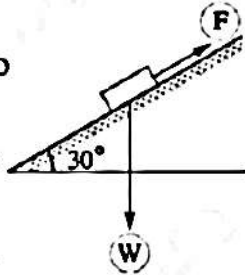


- (a) $22\sqrt{3}$ (b) $44\sqrt{3}$ (c) 22 (d) 44

- 10) In the opposite figure :
 A body of mass 5 kg. is placed on a rough inclined plane , connected with light string passes over a smooth pulley at the edge of the plane and the other end of the string tied to a body of mass 6 kg. If the system is in equilibrium , then the magnitude and the direction of the friction force is



- (a) 3.5 kg.wt. upward. (b) 3.5 kg.wt. downward.
 (c) 8.5 kg.wt. upward. (d) 8.5 kg.wt. downward.

11)	If a body is placed on a rough inclined plane and it was about to slide , then tangent of the friction angle is equal to each of the following except (a) the coefficient of friction. (b) the ratio between the normal reaction and the magnitude of the resultant reaction. (c) tangent of the inclination angle between the plane and the horizontal. (d) the ratio between the limiting friction and the magnitude of normal reaction.	
12)	A body of weight (W) N. is placed on a rough inclined plane. It makes an angle of measure (θ) to the horizontal. The coefficient of friction between the body and the plane equals (μ) , then the tangential force that acts on the body and makes the friction vanished equals N. (a) μW (b) $\mu \cos \theta$ (c) $\mu W \cos \theta$ (d) $W \sin \theta$	
13)	In the opposite figure : A body of weight (W) newton is placed on a rough plane inclined to the horizontal by an angle of measure 30° , a force of magnitude F newton in the direction of the line of the greatest slope upwards the plane acts on it to make the body about to move upwards when the magnitude of the resultant reaction between the body and the plane equals (W) newton , then $F = \dots\dots\dots$ newton.  (a) W (b) $\frac{\sqrt{3}}{2} W$ (c) $\frac{1}{2} W$ (d) $\sqrt{3} W$	
14)	A body of weight 4 kg.wt. is placed on a rough plane inclined to the horizontal at an angle of measure 45°. The coefficient of static friction between them is $\mu_s = \frac{1}{2}$, then the greatest force can preserve the body equilibrium acting in direction of the line of the greatest slope of the plane is kg.wt. (a) $3\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$	

15)

(2nd Session 2022) In the opposite figure :

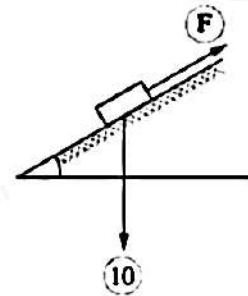
A body of weight 10 newton is placed on a rough plane inclined to the horizontal by an angle whose sine is $\frac{3}{5}$, the coefficient of the static friction between the body and the plane equals $\frac{1}{4}$, a force of magnitude F newton acts on the body in the direction of the line of the greatest slope upward to make it about to move upward, then F = newton.

(a) 8

(b) 10

(c) 6

(d) 4

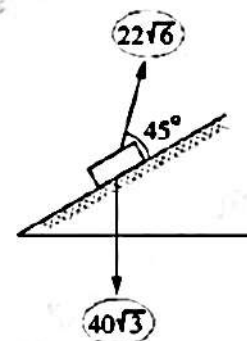


16)

In the opposite figure :

A body of weight $40\sqrt{3}$ kg.wt. is placed on a rough plane inclined to the horizontal by an angle whose tangent = $\frac{3}{4}$, a force of magnitude $22\sqrt{6}$ kg.wt. making an angle of measure 45° with the plane upwards acts on the body.

If the body is about to move downwards, then the coefficient of the static friction between the body and the plane =

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{4}$ 

17)

In the opposite figure :

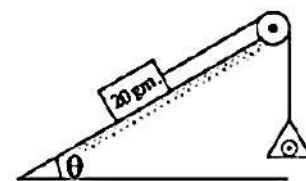
$\tan \theta = \frac{4}{3}$, and the scale pan mass is 7 gm. and the mass of the body on the inclined plane = 20 gm. The coefficient of static friction between the body and the plane equals $\frac{1}{6}$, then the mass should be on the scale pan to vanish the friction force = gm.

(a) 9

(b) 10

(c) 11

(d) 12



Second: Geometry and measurementUnit Three

LESSON 1: Straight lines and the plane.

LESSON 2: Pyramid and cone.

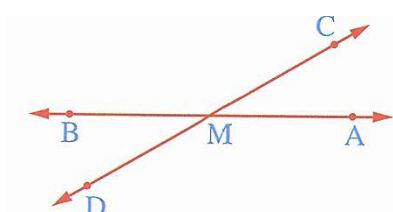
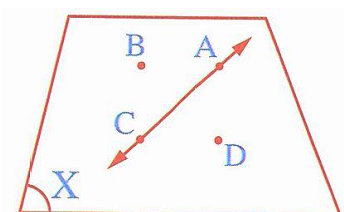
LESSON 3: Lateral area and total surface area of a pyramid and a cone.

LESSON 4: Volume of a pyramid and a right cone.

LESSON 5: Equation of a circle.

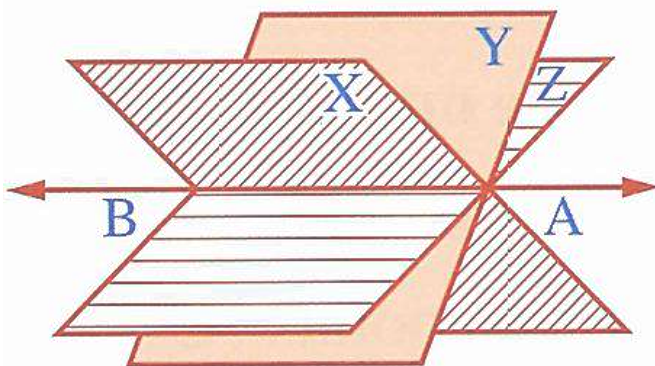
LESSON 1 : Straight lines and the plane.

Main Points

- The straight line: for any two points in the plane there is only one straight line passes through them.
- 
- The plane: is a surface with no ends such that a straight line joining any two points of its points lies wholly in its surface
- 
- The space: is the set of an infinite number of points, it contains all figures, planes and solids. the space contains four different non coplanar points at least.

Remarks

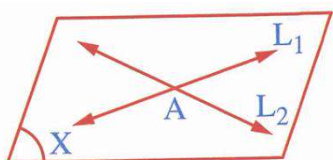
- Any point in the space passing through it an infinite number of the straight lines.
- Any point in the space passing through it an infinite number of the planes.
- Any two points in the space passing through them one and only one straight line.
- Any two points in the space passing through them an infinite number of the planes.



The relation between two
different straight lines in
the space

Intersected

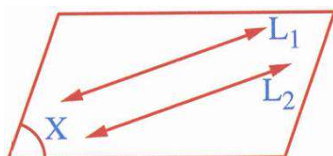
If they intersected
at only one point



$$L_1 \cap L_2 = \{A\}$$

Parallel

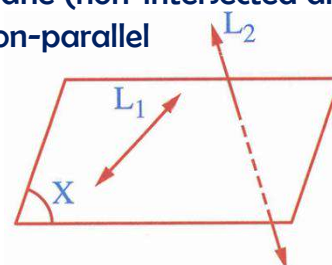
If they lie in the same plane
without intersection



$$L_1 \cap L_2 = \emptyset$$

Skew

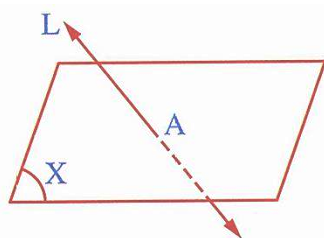
If they do not lie in the same
plane (non-intersected and
non-parallel)



$$L_1 \cap L_2 = \emptyset$$

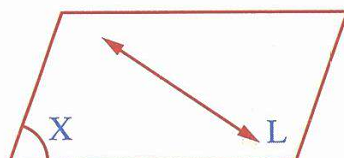
The relation between a straight
line and Plane in the space

The straight line cut the
plane at a point



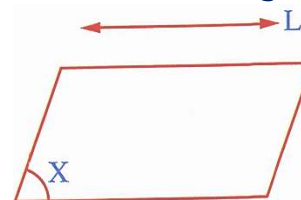
$$\text{i.e. } L \cap X = \{A\}$$

The straight line lies
completely in the plane.



$$\text{i.e. } L \cap X = L$$

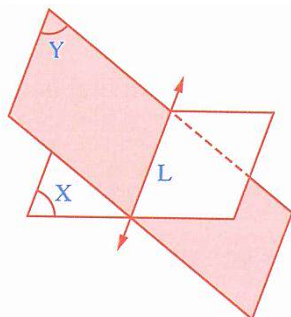
The straight line does not
intersect with the plane at any
point in this case the plane is
parallel to the straight line.



$$\text{i.e. } L \cap X = \emptyset$$

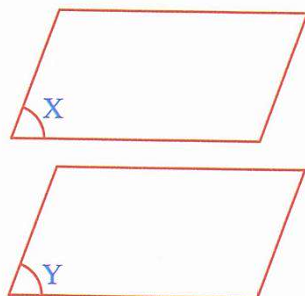
The relation between
two different planes in
the space

Intersected at a straight
line



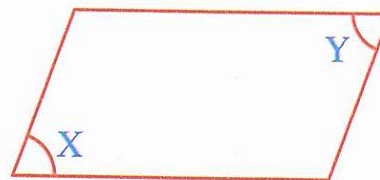
$$\text{i.e. } X \cap Y = L$$

Two parallel planes



$$\text{i.e. } X \cap Y = \emptyset$$

Two coincided planes.



$$\text{i.e. } X \cap Y = X = Y$$

Exercises (1)

[1] Complete each of the following:

[1] If the straight-line $L \parallel$ the plane X , then $L \cap X = \dots\dots\dots$

[2] If the straight-line $L \subset$ the plane X , then $L \cap X = \dots\dots\dots$

[3] If the straight line $L_1 \parallel$ the straight line L_2 , then $L_1 \cap L_2 = \dots\dots\dots$

[4] If X and Y are two planes such that: $X \cap Y = \varnothing$, then $X \dots\dots\dots Y$

[5] The two skew straight lines are neither nor

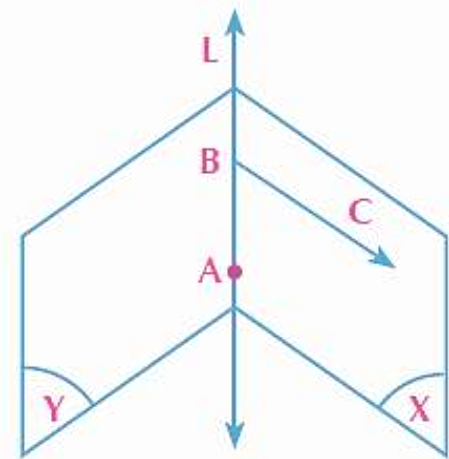
Answer the following questions:

[1] State the number of planes that passes through the following:

- (a) One given point
- (b) Two different points
- (c) Three collinear points
- (d) Three non-collinear points.
- (e) Four non-coplanar points

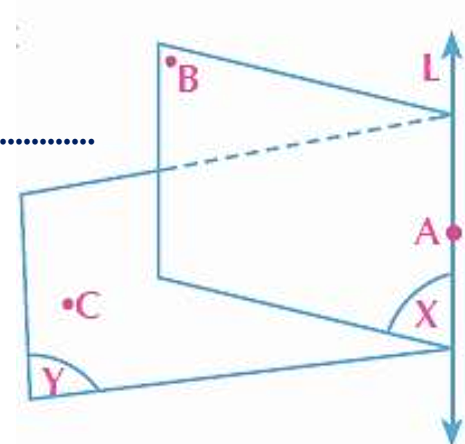
[2] Meditate the following figure, and then complete using one of the following symbols ($\in$, $\notin$, $\subset$, $\not\subset$)

- (a) L X
- (b) A X
- (c) C Y
- (d) $\overleftrightarrow{BC}$ Y

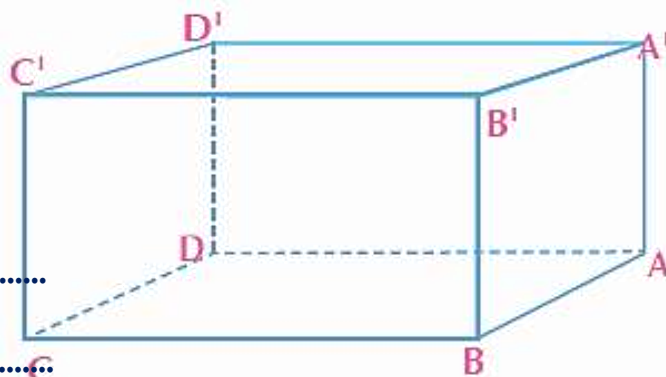


[3] In the opposite figure : X, Y are two planes intersected at the straight line $L, A \in L, B \in X, B \notin Y, C \in Y, C \notin X$ Complete the following:

- (a) The plane $X \cap$ The plane $ABC =$
- (b) The plane $Y \cap$ The plane $ABC =$
- (c) The plane $X \cap$ The plane $Y \cap$ The plane $ABC =$



[4] Meditate the following figure, and then complete the following:

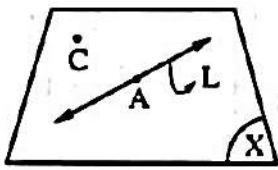
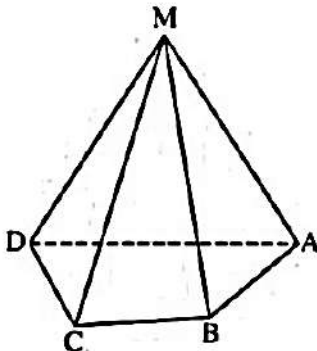
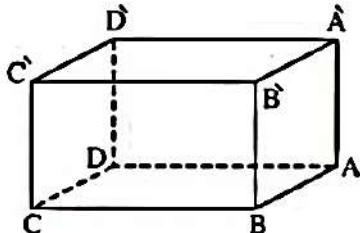
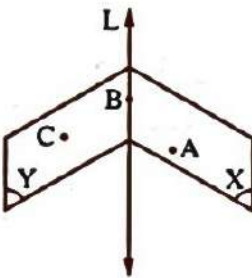


- (a) The plane ABCD // The plane
- (b) The plane BCC'B' // The plane
- (c) The plane ABB'A' $\cap$ The plane ABCD =
- (d) The plane ABB'A' $\cap$ The plane DCC'D' =
- (e) The plane DCC'D' $\cap$ The plane ABCD $\cap$ The plane ADD'A' =

Choose the correct answer:

1)	Number of straight lines that are passing through a given point is	
	(a) 1 (b) 2 (c) 3 (d) an infinite number.	
2)	Number of straight lines that are passing through two given points is	
	(a) 1 (b) 2 (c) 3 (d) an infinite number.	
3)	Number of planes that are passing through two given points is	
	(a) 1 (b) 2 (c) 3 (d) an infinite number.	
4)	Number of planes that are passing through three non-collinear points is	
	(a) 1 (b) 2 (c) 3 (d) an infinite number.	

5)	Number of planes that are passing through three collinear points is (a) Zero (b) 1 (c) 3 (d) an infinite number.	
6)	All of the following cases determine a plane except (a) a straight line and a point doesn't belong to it. (b) two parallel and not coincident straight lines. (c) two intersecting straight lines. (d) two skew straight lines.	
7)	Two skew straight lines are two straight lines which are (a) not intersect. (b) not perpendicular. (c) not parallel. (d) not intersect and not parallel.	
8)	The two straight lines are skew if they are (a) not parallel. (b) not intersecting. (c) not coincident. (d) not contained in the same plane.	
9)	 If the straight line $L //$ the plane X and $A \in X$, then $L \cap X = \dots\dots\dots$ (a) $\emptyset$ (b) L (c) X (d) $\{A\}$	
10)	 If the straight line $L \subset$ the plane X and $A \in X$, then $L \cap X = \dots\dots\dots$ (a) $\emptyset$ (b) L (c) X (d) $\{A\}$	
11)	If X, Y are two planes where $X \cap Y = \emptyset$, then $X \dots\dots\dots Y$ (a) $\perp$ (b) $//$ (c) $=$ (d) $\subset$	
12)	If $\overline{AB} \subset$ plane X , $\overline{CD} //$ plane X , then $\overline{CD}, \overline{AB}$ are (a) parallel only. (b) skew only. (c) parallel or skew. (d) intersecting.	

13)	The least number of planes that determine a solid is		
(a) 1	(b) 2	(c) 3	(d) 4
14)	Using the opposite figure , which of the following statements is not true ?		
(a) $L \subset X$	(b) $A \in L, A \notin X$	(c) $C \in X, C \notin L$	(d) $\overline{AC} \cap L = \{A\}$
15)	In the opposite figure : The plane $ABD \cap$ The plane $MCD = \dots\dots\dots$		
(a) $\overline{AM}$	(b) $\overline{CD}$	(c) $\{D\}$	(d) $\overline{MC}$
16)	In the opposite figure : The plane $A\hat{A}\hat{B} \cap$ the plane $\hat{A}\hat{C}\hat{C} = \dots\dots\dots$		
(a) $\overline{AA'}$	(b) $\overline{BB'}$	(c) $\overline{CC'}$	(d) $\overline{AC}$
17)	In the opposite figure : The plane $X \cap$ the plane $Y = \dots\dots\dots$		
(a) $\{B\}$	(b) $\{A, B, C\}$	(c) the straight line L	(d) $\emptyset$

18)

 In the opposite figure :

First : The plane $ABB\hat{A} \cap$ the plane $BC\hat{C}B = \dots\dots\dots$

- (a) $\overleftrightarrow{BB}$ (b) $\emptyset$ (c) $\{B\}$ (d) $\overleftrightarrow{AC}$

Second : The plane $ABC \cap$ the plane $\hat{A}B\hat{C} = \dots\dots\dots$

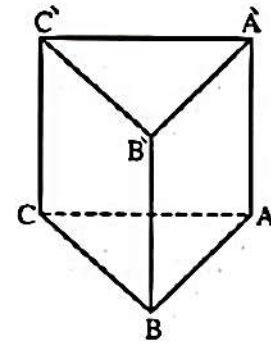
- (a) $\overleftrightarrow{BB}$ (b) $\emptyset$ (c) $\overleftrightarrow{AA}$ (d) $\overleftrightarrow{AB}$

Third : $\overleftrightarrow{AC} \cap \overleftrightarrow{AC} = \dots\dots\dots$

- (a) $\{A\}$ (b) $\{C\}$ (c) $\overleftrightarrow{AA} \cap \overleftrightarrow{BB}$ (d) $\overleftrightarrow{AC}$

Fourth : $\overleftrightarrow{BB} \cap$ the plane $ABC = \dots\dots\dots$

- (a) $\overleftrightarrow{BB}$ (b) $\{B\}$ (c) $\{B\}$ (d) $\emptyset$



19)

 In the opposite figure :

First : The plane $MAB \cap$ the plane $MBC = \dots\dots\dots$

- (a) $\overleftrightarrow{AB}$ (b) $\overleftrightarrow{MB}$ (c) $\emptyset$ (d) $\{M\}$

Second : The plane $MBC \cap$ the plane $ABC = \dots\dots\dots$

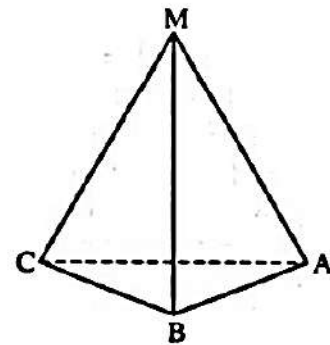
- (a) $\{B\}$ (b) $\emptyset$ (c) $\overleftrightarrow{AB}$ (d) $\overleftrightarrow{BC}$

Third : $\overleftrightarrow{MB} \cap$ the plane $ABC = \dots\dots\dots$

- (a) $\overleftrightarrow{MB}$ (b) $\emptyset$ (c) $\{B\}$ (d) $\{M\}$

Fourth : The plane $MAB \cap$ the plane $MBC \cap$ the plane $MAC = \dots\dots\dots$

- (a) $\overleftrightarrow{MB}$ (b) $\overleftrightarrow{MC}$
(c) the solid $MABC$ (d) $\{M\}$



20)

 In the opposite figure :

X and Y are two intersecting planes at the straight line L

, $A \in L$, $B \in X$, $B \notin Y$, $C \in Y$, $C \notin X$:

First : The plane $X \cap$ the plane ABC =

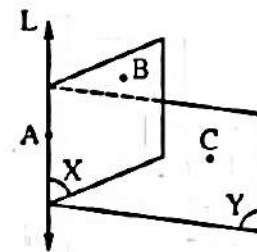
- (a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) L

Second : The plane $Y \cap$ the plane ABC =

- (a) $\overline{AB}$ (b) $\{A\}$ (c) $\overline{CB}$ (d) $\overline{AC}$

Third : The plane $X \cap$ the plane $Y \cap$ the plane ABC =

- (a) $\emptyset$ (b) L (c) $\{A\}$ (d) $\{B\}$



21)

In the opposite figure :

First : The plane ABCD // the plane

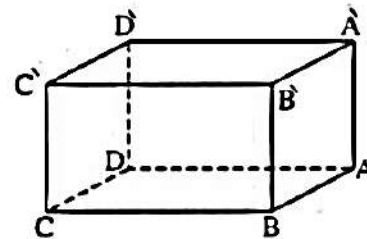
- (a) $\overline{ABC}$ (b) $\overline{A'B'D'}$
(c) $\overline{ABB'}$ (d) $\overline{A'B'C'}$

Second : The plane $BCC'B' //$ the plane

- (a) ABC (b) $\overline{A'B'C'}$ (c) ABD (d) $\overline{A'A'D'}$

Third : The plane $\overline{ABB'A'} \cap$ the plane ABCD =

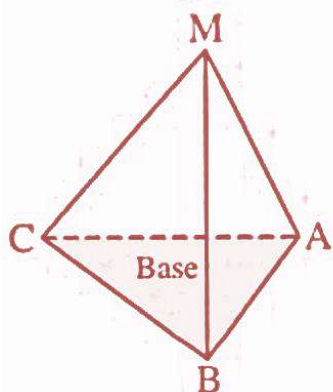
- (a) $\{B\}$ (b) $\{A, B\}$ (c) $\overline{AB}$ (d) $\overline{A'B'}$



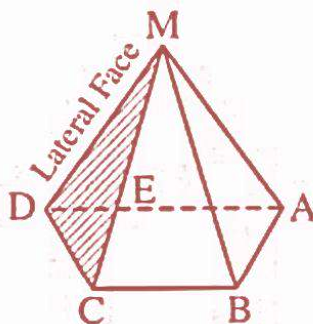
Lesson 2: The pyramid

Main Points

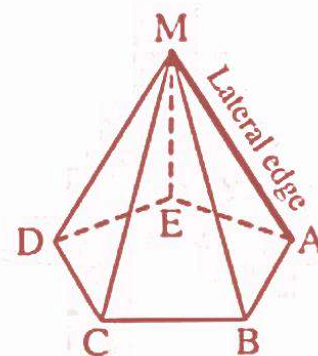
- **Net of a solid:** It's a two-dimensional figure that can be folded to be a three-dimensional figure.
- **Pyramid:** It's a solid which has a single base and all of its other faces are triangles that have a common vertex. The pyramid is named according to the number of sides of its polygon base. It could be a triangular pyramid, quadrangular pyramid, pentagonal pyramid .. and so on



triangular pyramid,
its base is a triangle

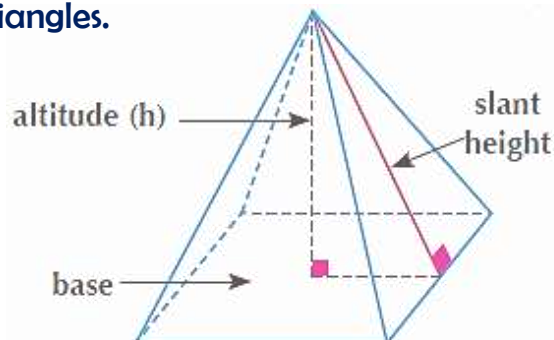


Quadrilateral pyramid,
its base is a quadrilateral



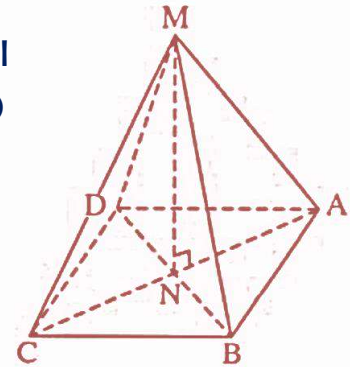
Pentagonal pyramid,
its base is a pentagon

- **Regular pyramid:** It's a pyramid whose base is a regular polygon. It's center is a foot of the perpendicular drawn from the vertex to the base and we find that:
 - ❖ Its lateral edges are equal in length.
 - ❖ Its lateral faces surfaces are isosceles and congruent triangles.
 - ❖ The slant heights are equal.



- **The right pyramid:** The pyramid is right if and only if, the foot of the perpendicular drawn from the vertex to its base is the geometrical center of the base.

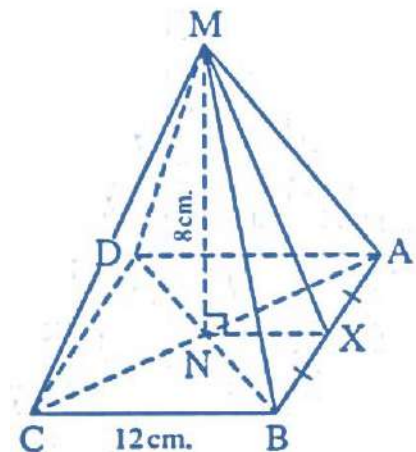
In the pyramid $MABCD$ as in the figure: If N is the geometrical center of the base $ABCD$ and $\overline{MN} \perp$ the plane of the base $ABCD$, then the pyramid $MABCD$ is called a right pyramid.



Remarks:

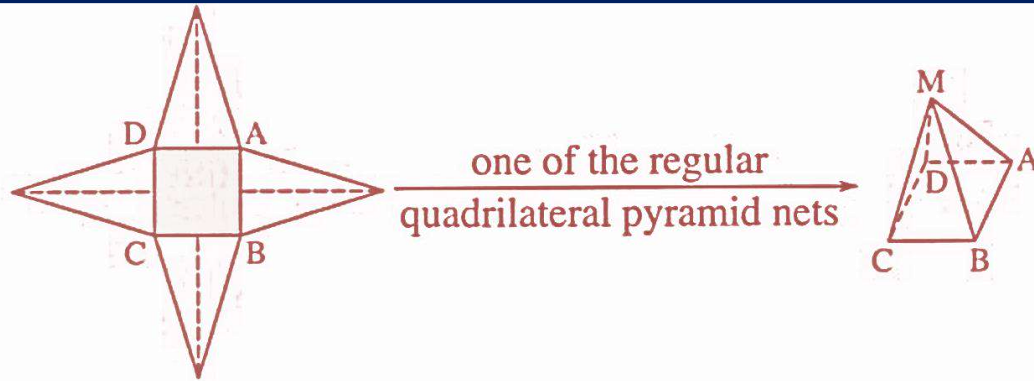
- . Every regular pyramid is a right pyramid but not vice versa.
- . Not necessary that the lateral edges of the right pyramid are equal in length.
- . Not necessary that the slant heights of the right pyramid are equal in lengths.
- . The regular triangular pyramid is called a triangular pyramid of regular faces if it's all faces are equilateral triangles and any one of them is its base.

Ex (1): $MABCD$ is a regular quadrilateral pyramid? the length of its base side is 12 cm. and its height length equals 8 cm. Find the length of its slant height.



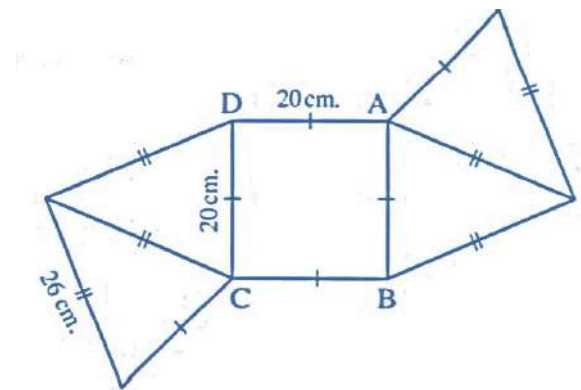
Solids net:

The solids net is used to make the solids by tracking the shape of the solid on the surface of the plane and folding this plane to form the solid.

The pyramid net:

Ex (2): The opposite net represents a net of a regular quadrilateral pyramid. Find:

- (1) Height of the pyramid.
- (2) The slant height of the pyramid.



The lateral area of regular pyramid. the total area of pyramid - the volume of pyramid:

- (1) The lateral area of the pyramid = the sum of areas of the lateral faces
- (2) The lateral area of the regular pyramid = $\frac{1}{2}$ base perimeter x slant height
- (3) The total surface area of the pyramid = lateral area + area of the base
- (4) The volume of the pyramid = $\frac{1}{3}$ base area x height

Ex (3): A regular quadrilateral pyramid the length of its base diagonal is $60\sqrt{2}$ cm. and its slant height is 50 cm. Find:

- (1) The height of the pyramid.
- (2) L.S.A and T.S.A of the pyramid.
- (3) Volume of the pyramid.

Exercises (2)

Answer the following questions

(1) In the regular pentagonal pyramid:

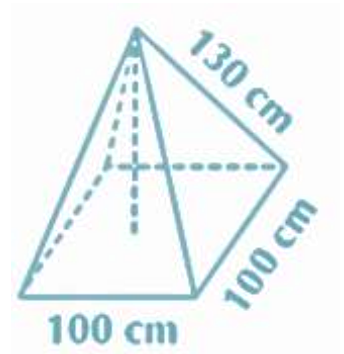
1. What the number of its lateral faces?
2. What the number of its faces?
3. What the number of its lateral edges?
4. What the number of its edges?

(2) In the regular pyramid, arrange the following lengths ascendingly

1. The length of the lateral edge
2. The height of the pyramid
3. The slant height

(3) The opposite figure shows a regular quadrangular pyramid.

Using the given data, find each of the slant height and the height



- (4) MABCD is a regular quad. pyramid its height 20 cm, and its slant height 25 cm.

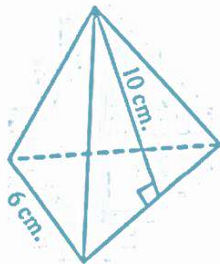
Find the length of the side of the pyramid's base.

(5) MABCD is a regular quadrilateral pyramid of height 20 cm. and slant height 25 cm.

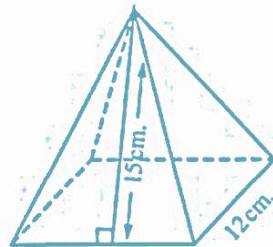
Find the length of its base side.

(6) Find the lateral area and the total area of each regular pyramid of the following:

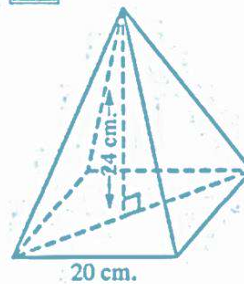
(1) 



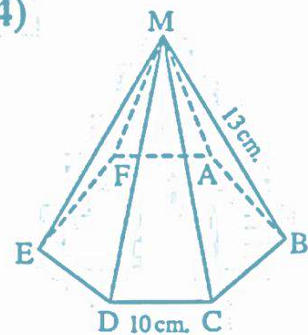
(2) 



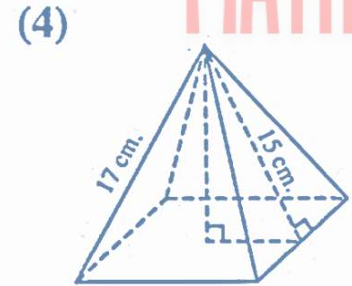
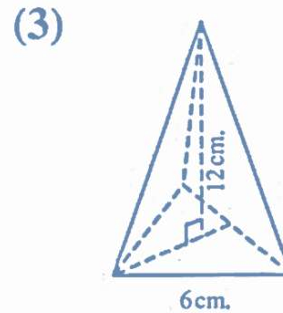
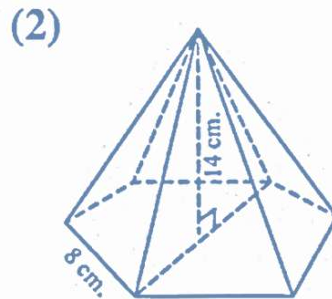
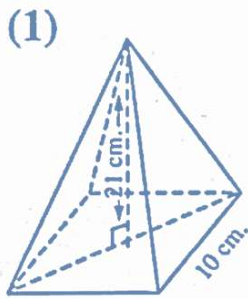
(3) 



(4)



(7) Find the volume of each of the following regular pyramids:



(8) The great pyramid of Giza (Khopo pyramid) is a regular quadrilateral pyramid the side length of its base is 232 metres and its slant height is 186 metres. Find height of the pyramid.

(9) A regular quadrilateral pyramid the length of its base is 20 cm. , and its height is $10\sqrt{3}$ cm.

Find : (1) Its lateral surface area.

(2) Its volume.

(10) Calculate to the nearest tenth the volume of a regular pentagonal pyramid whose side length of its base = 16 cm. and its height = 12 cm.

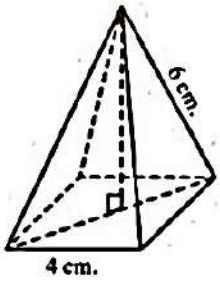
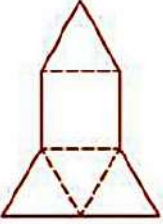
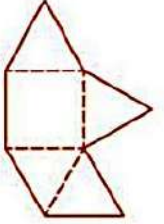
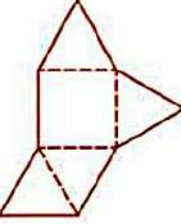
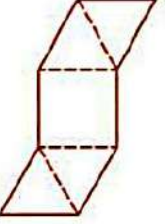
(11) A regular hexagonal pyramid, the side length of its base = 12 cm. and its slant height = $10\sqrt{3}$ cm.,

Find: (1) Its lateral area.

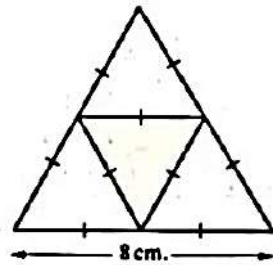
(2) Its total area.

Choose the correct answer:

1)	The line segment joining the vertex of the pyramid and any vertex of its base vertices is called (a) the height of the pyramid. (b) the slant height of the pyramid. (c) the lateral edge of the pyramid. (d) the side of its base.	
2)	Which of the following statements is true ? (a) The lateral faces of the right pyramid are congruent. (b) The regular pyramid is a right pyramid. (c) The heights of the lateral faces of the right pyramid are equal. (d) The least number of planes that can determine a solid = 3 planes.	
3)	In the regular pyramid , which of the following is the right ascendingly order of the lengths ? (a) The length of the lateral edge , the height , the slant height. (b) The height , the slant height , the length of the lateral edge. (c) The slant height , the height , the length of the lateral edge. (d) The length of the lateral edge , the slant height , the height.	
4)	The shape of the base of a regular pyramid must be (a) parallelogram. (b) rhombus. (c) rectangle. (d) square.	
5)	The number of all faces of a regular pentagonal pyramid is (a) 5 (b) 6 (c) 7 (d) 10	
6)	If the number of the faces of a pyramid = m and the number of its vertices = n , then the number of its edges = (a) $m + n$ (b) $m + n - 1$ (c) $m + n - 2$ (d) $m + n + 2$	

7)	In the hexagonal pyramid : number of faces + number of vertices – number of edges = (a) 1 (b) 2 (c) 3 (d) 4	
8)	The opposite figure represents a regular quadrilateral pyramid of height = cm. (a) $7\sqrt{2}$ (b) $2\sqrt{7}$ (c) $4\sqrt{2}$ (d) $2\sqrt{5}$	
9)	A regular quadrilateral pyramid , if the length of its base side is 6 cm. , the length of its lateral edge is 8 cm. , then the length of its height = cm. (a) $5\sqrt{2}$ (b) $\sqrt{46}$ (c) $\sqrt{85}$ (d) $\sqrt{48}$	
10)	 Which of the following nets does not make a regular quadrilateral pyramid when it folded ?  (a)  (b)  (c)  (d)	
11)	The ratio between the edge length of the triangular regular faces pyramid : its height = (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 2$ (c) $\sqrt{6} : 2$ (d) $\sqrt{3} : 3$	
12)	The ratio between the length of the edge of the regular faces pyramid : its slant height = (a) $2\sqrt{2} : \sqrt{3}$ (b) $2\sqrt{3} : 3$ (c) $\sqrt{6} : 2$ (d) $\sqrt{6} : 3$	

13)	A right quadrilateral pyramid of height 10 cm. , its base is a rhombus whose diagonal lengths are 12 cm. and 8 cm. , then its volume = cm ³ (a) 40 (b) 80 (c) 160 (d) 200	
14)	 A regular quadrilateral pyramid whose base perimeter is 36 cm. , and its height is 10 cm. , then its volume = cm ³ (a) 810 (b) 180 (c) 360 (d) 270	
15)	A regular pyramid whose volume is 12 cm ³ , and its base area is 4 cm ² , then its height = cm. (a) 3 (b) 6 (c) 9 (d) 2	
16)	A regular quadrilateral pyramid whose volume 64 cm ³ , and its height is 6 cm. , then its base perimeter = cm. (a) 8 (b) $8\sqrt{2}$ (c) 16 (d) $16\sqrt{2}$	
17)	In a regular quadrilateral pyramid , the length of its base is 10 cm. , the length of its slant height is 13 cm. , then its lateral area equal cm ² (a) 260 (b) 360 (c) 130 (d) 520	
18)	A regular quadrilateral pyramid , the area of its base = 100 cm ² and its height is 12 cm. , then its lateral area equal cm ² (a) 260 (b) 520 (c) 130 (d) 360	
19)	The total area of a right quadrilateral pyramid , its base is a regular polygon and its diagonal length = $10\sqrt{2}$ cm. and its height = $5\sqrt{3}$ cm. equals cm ² (a) 40 (b) 100 (c) 200 (d) 300	
20)	A triangular regular faces pyramid , its edge length 10 cm. , then its total area equal cm ² (a) 40 (b) 100 (c) $100\sqrt{3}$ (d) $25\sqrt{3}$	

21)	If the total area of a regular faces pyramid = $36\sqrt{3} \text{ cm}^2$, then the sum of its edges lengths = cm. (a) 6 (b) 12 (c) 18 (d) 36	
22)	A regular triangular pyramid, its base length is $6l \text{ cm}$. and its height $l \text{ cm}$. , then its lateral area = cm^2 (a) $27\sqrt{3}l^2$ (b) $18l^2$ (c) $9\sqrt{3}l^2$ (d) $36l^2$	
23)	A triangular regular faces pyramid, if the sum of the lengths of its edges equal 18 cm. , then its volume = cm^3 (a) $9\sqrt{2}$ (b) $\frac{9\sqrt{2}}{4}$ (c) $\frac{27\sqrt{2}}{5}$ (d) $9\sqrt{3}$	
24)	A regular quadrilateral pyramid whose total area = 70 cm^2 and its lateral area = 45 cm^2, then its height = cm. (a) 2.5 (b) 5 (c) $\sqrt{14}$ (d) 4.5	
25)	In a regular quadrilateral pyramid the length of its base side 10 cm. and the area of one of its lateral faces 60 cm^2, then its total area equal cm^2 (a) 600 (b) 340 (c) 160 (d) 240	
26)	The ratio between the lateral surface area of a triangular pyramid of regular faces to the area of its total surface area = (a) 1 : 3 (b) 1 : 4 (c) 3 : 4 (d) 1 : 2	
27)	By using the opposite solid net : The lateral area of the resulted solid = cm^2 (a) $8\sqrt{3}$ (b) $12\sqrt{3}$ (c) $16\sqrt{3}$ (d) 24	

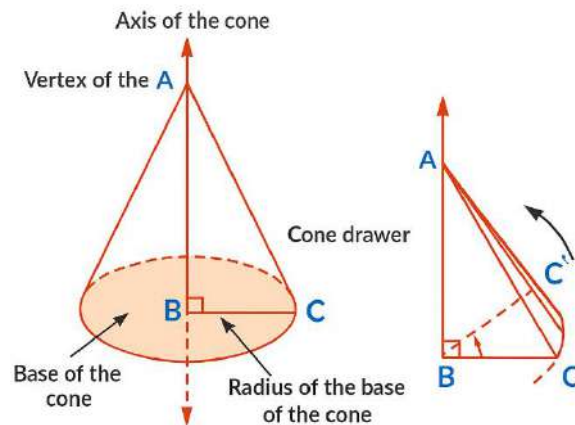
Lesson 3: The cone

The right circular cone:

Is the solid formed from the rotation of a right- angled triangle with complete rotation about one of its right sides as an axis or is the space formed from the folding of a circular sector where their two radii coincide on each other.

In the opposite figure:

$\triangle ABC$ is a right-angled triangle at B . if it is rotated about the axis $\overleftrightarrow{AB}$ with a full turn, the formed solid is called right circular cone. and the point A is called vertex of the cone, $\overleftrightarrow{AC}$ is the drawer of the cone, $\overleftrightarrow{AB}$ is axis of the cone, surface of circle B is the base of the cone.



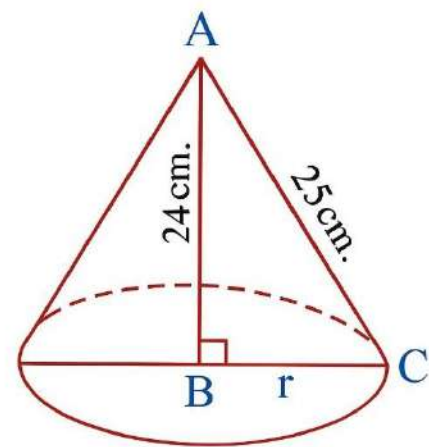
Example (1)

A right circular cone 5 the length of its drawer is 25 cm. and its height is 24 cm.

Find the perimeter and the area of

the base of the cone. ($\pi = \frac{22}{7}$)

sol.



$$\therefore \overleftrightarrow{AB} \perp \text{the plane of circle B} \quad \therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$$

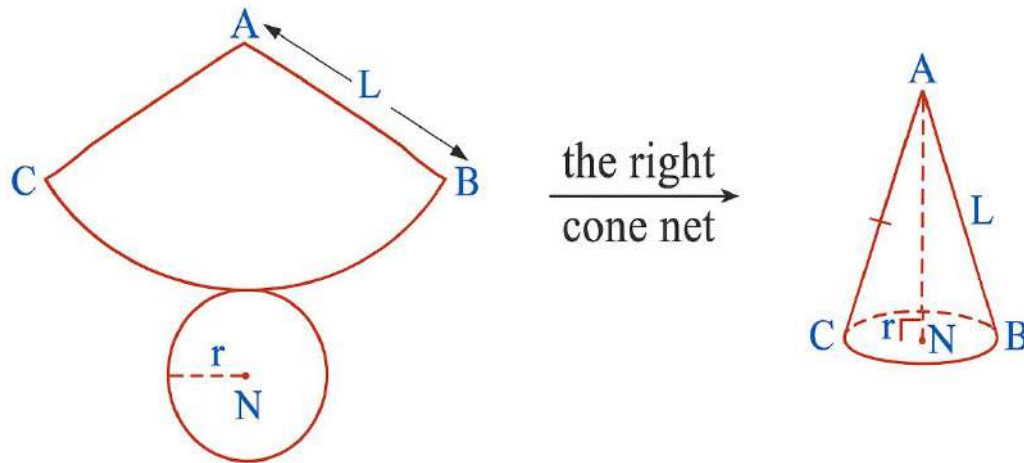
$$\therefore m(\angle ABC) = 90^\circ \quad \therefore (BC)^2 = (AC)^2 - (AB)^2 = (25)^2 - (24)^2 = 49$$

$$\therefore BC = 7 \text{ cm.} \quad \therefore r \text{ (the radius of the base)} = 7 \text{ cm.}$$

$$\therefore \text{The perimeter of the base} = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

$$\therefore \text{the area of the base} = \pi r^2 = \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

• **The right cone net:**



Note:

(The area of the sector = $\frac{1}{2}r^2\theta^{\text{rad}}$, r radius of circular sector, θ^{rad} is the measure of the angle in radian).

Example (2)

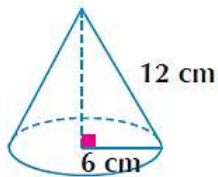
A piece of cardboard is folded in the form of circular sector the length of its radius 36 cm and the measure of its angle 210° for making a right circular cone has the greatest area. Find the height of the cone.

The lateral area - total area - volume of a right cone:

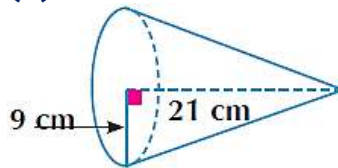
- Lateral area of the right cone = $\pi L r$, where L is the length of the slant height, r is the length of its base radius.
- Total area of the right cone = $\pi L r + \pi r^2 = \pi r (L + r)$

Find lateral area, and total area for each right cone, according to the given data.

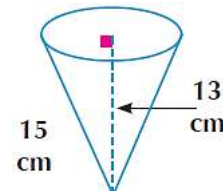
(a)



(b)



(c)

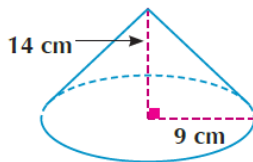


Find the length of the radius of a right cone, if the length of the cone drawer 15 cm and its total area $154 \pi \text{ cm}^2$

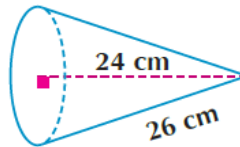
- Volume of the right cone = $\frac{1}{3}$ area of the base $\times$ its height.

Find the volume of the right cone shown in the figure using the given data.

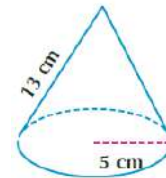
(a)



(b)



(c)



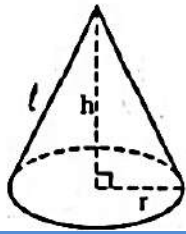
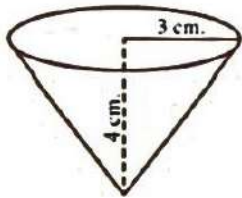
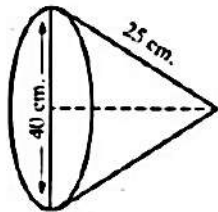
Answer the following questions

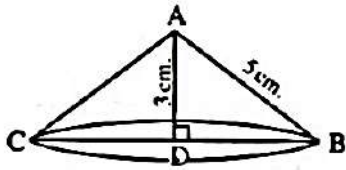
- [1] Which is greater in volume? A right cone the length of its base radius is 15 cm, and its height is 20 cm, or a regular quadrangular pyramid its height is 40 cm, and the perimeter of its base is 48 cm.

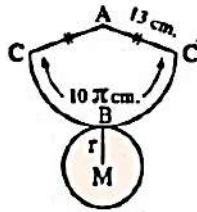
[2] Find the volume of a right cone its base perimeter is 44 cm, and its height is 25 cm.

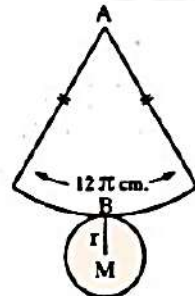
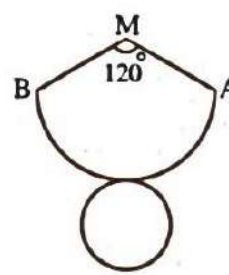
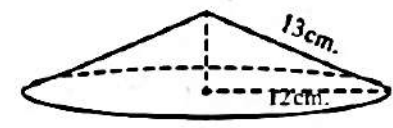
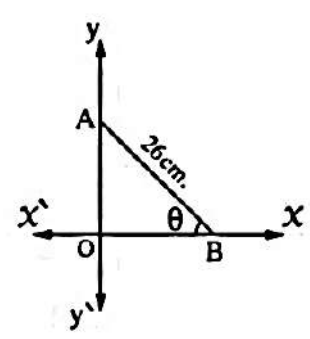
[3] Find the volume of a right cone its lateral area is 220 cm^2 , and the length of its
drawer is 14 cm.

Choose the correct answer:

1)	The right circular cone is generated by folding a paper in the shape of	
	(a) an equilateral triangle. (b) a right-angled triangle. (c) a circular segment. (d) a circular sector.	
2)	The measure of the smallest rotation angle of an isosceles triangle around its axis of symmetry to form a right circular cone is	
	(a) 90° (b) 180° (c) 270° (d) 60°	
3)	If a right circular cone intersected by a plane parallel to its base , then the resulted sector is	
	(a) an isosceles triangle. (b) an equilateral triangle. (c) a circle. (d) a trapezoid.	
4)	The total area for the opposite right cone equals	
	(a) $\pi r l$ (b) $\frac{\pi}{3} \pi^2 h$ (c) $\pi r (r + l)$ (d) $\frac{\pi}{3} r (r h + 3 l)$	
5)	In the opposite figure : The length of the drawer = cm.	
	(a) 2 (b) 3 (c) 4 (d) 5	
6)	In the opposite figure : The height of the cone = cm.	
	(a) 15 (b) 20 (c) 25 (d) 40	

7)	In a right circular cone , if the length of its height 15 cm. , and the length of its drawer 17 cm. , then its radius length equal cm. (a) 10 (b) 8 (c) 7 (d) 9	
8)	In a right circular cone , the radius length of its base = 15 cm. and its height = 20 cm. , then its lateral area = cm ² (a) 375π (b) 600π (c) 1500π (d) 1875π	
9)	In the opposite figure : If AD = 3 cm. , AB = 5 cm. , then the total area of the cone = cm ² (a) 8π (b) 24π (c) 48π (d) 36π	
10)	The height of a right circular cone is 6 cm. and the circumference of its base is 16π cm. , then its lateral area = cm. ² (a) 144π (b) 64π (c) 60π (d) 80π	
11)	A right circular cone , the length of its drawer equal the length of the diameter of its base , then its total area = cm ² (a) $3 \pi r^2$ (b) $3 \pi r^3$ (c) $4 \pi r^2$ (d) $4 \pi r^3$	
12)	 A right circular cone , its height 24 cm. , and the length of its drawer 26 cm. , then the area of its base cm ² (a) 25π (b) 100π (c) 20π (d) 50π	
13)	 The radius length of the base of a right circular cone where its total area = 616π cm ² , and the length of its drawer is 30 cm. is cm. (a) 44 (b) 14 (c) 30 (d) 34	

14)	 Lamp cover is in the form of a right circular cone , the circumference of its base circle = 88 cm. , its height = 20 cm. , then its lateral area $\simeq$ cm^2 ($\pi = \frac{22}{7}$) (a) 88 (b) 596 (c) 1074 (d) 1047	
15)	A right circular cone , the radius length of its base = 6 cm. and the length of its drawer = 10 cm. , then its volume = cm^3 (a) 32π (b) 64π (c) 96π (d) 228π	
16)	A right circular cone where its height 4 cm. , the length of its drawer 5 cm. , then its volume cm^3 (a) 36π (b) 15π (c) 24π (d) 12π	
17)	A right circular cone where its volume $27\pi \text{ cm}^3$, and the circumference of its base $6\pi \text{ cm}$. , then its height equal cm. (a) 27 (b) 18 (c) 9 (d) 6	
18)	A right circular cone , the radius length of its base 5 cm. and its total area = $90\pi \text{ cm}^2$, then its volume = cm^3 (a) 105π (b) 95π (c) 100π (d) 120π	
19)	The volume of a right circular cone , if the length of its drawer = 15 cm. and its total area = $216\pi \text{ cm}^2$ equal cm^3 (a) 205π (b) 220π (c) 280π (d) 324π	
20)	If the volume of a right circular cone is $9\pi \text{ cm}^3$ and the length of its base radius equal the length of its height , then its base area = cm^2 (a) 9π (b) 3π (c) 27π (d) 12π	
21)	The opposite net describes a solid its volume = cm^3 (a) 25π (b) 50π (c) 75π (d) 100π	

22)	The opposite net describes a solid its volume = $96 \pi \text{ cm}^3$, then its total area = cm^2 (a) 16π (b) 32π (c) 48π (d) 96π	
23)	The opposite figure represents the net of a solid , MB = $3 \pi \text{ cm}$, $m(\angle AMB) = 120^\circ$, then the volume of the solid = cm^3 (a) $2\sqrt{2} \pi^2$ (b) $\frac{2\sqrt{2}}{3} \pi^4$ (c) $2\sqrt{2} \pi$ (d) $\frac{2\sqrt{2}}{3} \pi^3$	
24)	The central angle of the sector if be folded it becomes the opposite cone is (a) acute. (b) obtuse. (c) straight. (d) reflex.	
25)	The area of a circular sector : the total area of the circular solid cone which can be formed from folding this sector (a) > 1 (b) < 1 (c) $= 1$ (d) ≥ 1	
26)	In the opposite figure : If $\tan \theta = \frac{5}{12}$, AB = 26 cm. , then the lateral area of the solid generated by rotating the triangle ABO a complete revolution about the X-axis = $\pi \text{ cm}^2$ (a) 360 (b) 260π (c) 260 (d) 360π	

Lesson 4: The circle

Main Points

- **The circle:** a set of plane points which are equidistant from a fixed point in the plane
- **Equation of a circle** = the equation of the circle whose centre the point (d , h) and the length of its radius equal r is $(x - d)^2 + (y - h)^2 = r^2$.
- **The general form of the equation of a circle** its centre is the point (-L , -K), and the length of its radius r is: $x^2 + y^2 + 2Lx + 2Ky + C = 0$, where $r = \sqrt{L^2 + K^2 - C}$, $L^2 + K^2 - C > 0$

To find the coordinates of the centre of the circle and the length of its radius from the general form of its equation:

- ❖ First verify to put the equation in the general form where the coefficient of x^2 = coefficient of y^2 = unity.
- ❖ The coordinates of the centre (-L , - k) = $\left(\frac{-\text{coefficient of } x^2}{2}, \frac{-\text{coefficient of } y^2}{2} \right)$
- ❖ The length of the radius of the circle equal $r = \sqrt{L^2 + K^2 - C}$, where $L^2 + K^2 - C > 0$

[1] Write the equation of a circle if its centre:

(A) M (4 , -3) , the length of its radius equals 5 units.

(B) M (7 , -1) , the length of its diameter equals 8 units.

(C) M (2 , 0) , the length of its diameter equals 28 units.

(D) M (0 , -5) , and passes through the point A(-2 , -9)

(E) The origin point, the length of its radius equals r units.

[2] Write the equation of the circle if:

(A) Its centre is the point M (-2 , 7), and passes through the point A (2 , 10).

(B) Its centre is the point M (5 , 4), and touches the straight line $x = 2$

(C) Its centre M lies in the 1st quad. of the coordinates plane where its radius is of length 3 units, and the two straight lines $x = 1$, $y = 2$ are two tangents to it.

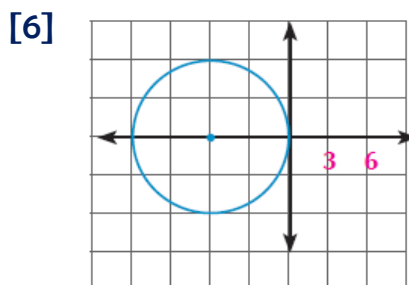
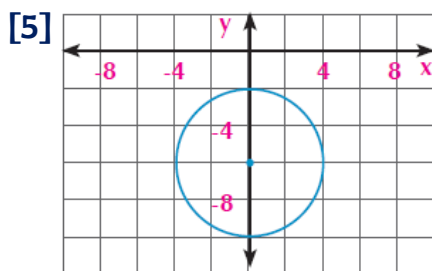
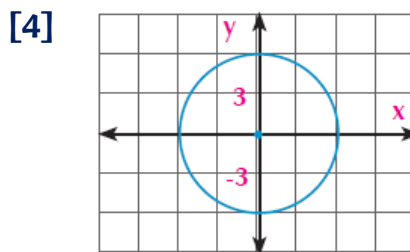
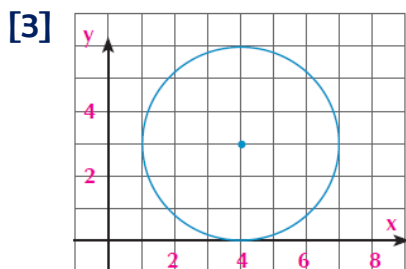
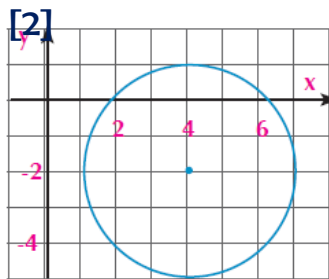
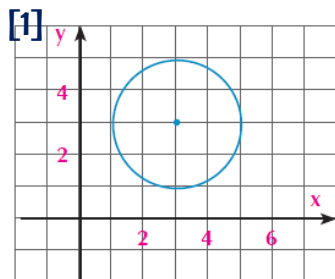
[3] Find the equation of the circle if:

(A) Its centre M (7 , -5), and it passes through the point A(3 , 2).

(B) $\overline{AB}$ is a diameter in the circle, where A(6 , -4) , B(0 , 2).

(C) Its centre is the point (5 , -3) and touches the x - axis

[4] Write the equation of a circle represented by the given figure:



[5] Find the centre, and the length of the radius for each of the following circles:

(A) $x^2 + y^2 = 27$

(B) $(x + 3)^2 + (y - 5)^2 = 49$

(C) $(x - 2)^2 + y^2 = 16$

(D) $x^2 + (y + 7)^2 = 24$

[6] Find the centre, and the length of the radius for each of the following circles:

(A) $x^2 + y^2 - 4x + 6y - 12 = 0$

(B) $x^2 + y^2 + 2x = 8$

(C) $x^2 + y^2 - 6x + 10y = 0$

(D) $x^2 + y^2 - 8x = 12$

[7] Write the general form of the equation of a circle in the following cases:

(A) Its centre M (3 , 1) ,and the length of its diameter equal 8.

(B) Its centre M (0 , 0), and it passes through the point A (-1 , 3)

(C) Its centre M (-5 , 0), and it passes through the point B (3 , 4)

(D) $\overline{AB}$ is a diameter in it, where A(3 , -7) , B (5 , 1)

[8] Show which of the following circles are congruent:

(A) $x^2 + y^2 - 2x + 4y - 3 = 0$

,

$x^2 + y^2 + 6x - 11 = 0$

(B) $x^2 + y^2 - 14x + 37 = 0$

,

$x^2 + y^2 + 10x + 13 = 0$

[9] Show which of the following equation is for a circle, then find its centre and the length of its radius:

(A) $x^2 + y^2 + 8x - 16y - 1 = 0$

(B) $x^2 + 2y^2 + 6x - 5y = 0$

(C) $\frac{1}{4}x^2 + \frac{1}{4}y^2 + x - 8 = 0$

(D) $x^2 + y^2 + 2xy - 12 = 0$

(E) $x^2 + y^2 - 2x + 4y + 7 = 0$

(F) $2x^2 + 2y^2 + 3y - 8 = 0$

Choose the correct answer:

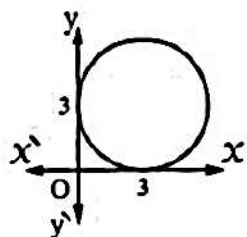
1)	The centre of the circle in which its diameter is $\overline{AB}$ where $A = (-1, 3)$, $B = (5, -3)$ is	
	(a) (4, 0) (b) (2, 0) (c) (-6, -6) (d) (0, 4)	
2)	The radius length of the circle whose equation $x^2 + y^2 - 4x + 2y - 4 = 0$ is length unit.	
	(a) 2 (b) 4 (c) 3 (d) 9	
3)	The radius length of the circle whose equation $(x+2)^2 + y^2 + 2y = 0$ is length unit.	
	(a) zero (b) 1 (c) 2 (d) 4	
4)	The diameter length of the circle : $4x^2 + 4y^2 + 16x - 8y - 16 = 0$ equals length unit.	
	(a) 3 (b) 6 (c) 12 (d) 24	
5)	If the two straight lines $y = -6$, $y = 8$ are two tangents to the circle M, then its radius length = length unit.	
	(a) 1 (b) 2 (c) 7 (d) 14	
6)	If the straight line $y = 2$ touches the circle M whose center is (6, 9), then its diameter length = length unit.	
	(a) 6 (b) 7 (c) 14 (d) 15	
7)	The radius length of the circle $(n+3)x^2 + y^2 - 4y + (m-2)xy + (m-n)x - 8 = 0$ is length unit.	
	(a) 2 (b) 4 (c) 6 (d) $2\sqrt{2}$	

8)	The area of the circle whose equation is $(x-5)^2 + (y+4)^2 = 7$ equals square unit. (a) 3.5π (b) 7π (c) 12.25π (d) 49π	
9)	The circumference of the circle whose equation $x^2 + y^2 + 2x - 2y - 2 = 0$ is length unit. (a) π (b) 2π (c) 4π (d) 8π	
10)	 The circumference of the circle whose equation is $x^2 + y^2 = 8$ is length units. (a) 8π (b) 64π (c) $2\sqrt{2} \pi$ (d) $4\sqrt{2} \pi$	
11)	If the two straight lines : $x = -3$, $x = 4$ touch the circle M , then its circumference = length units. where $(\pi = \frac{22}{7})$ (a) 22 (b) 44 (c) 12 (d) 14	
12)	The equation : $\begin{vmatrix} x & y \\ y & x \end{vmatrix} - 49 = 0$ represents the equation of a circle with radius length length unit. (a) 49 (b) 14 (c) 9 (d) 7	
13)	Which of the following equations represent a circle ? (a) $x^2 - y^2 + x - y = 6$ (b) $2x^2 + y^2 - x + y = 5$ (c) $x^2 + y^2 - x = 6$ (d) $x^2 + y^2 - xy = 6$	
14)	If the equation : $2x^2 + (a-1)y^2 + 5x - 3y = 7$ represent a circle , then a = (a) 1 (b) 2 (c) 3 (d) 4	

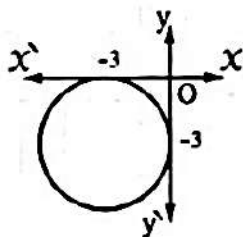
15)	The center of the circle whose equation $(X - 2)^2 + (y + 3)^2 = 16$ is	(a) (2 , 3)	(b) (2 , - 3)	(c) (13 , 16)	(d) (4 , 9)
16)	The centre of the circle whose equation $2x^2 + 2y^2 + 12x - 16y = 0$ is	(a) (3 , - 4)	(b) (- 6 , 8)	(c) (- 3 , 4)	(d) (6 , - 8)
17)	 The circle $(x + 2)^2 + y^2 + 2y = 0$ its centre is the point	(a) (2 , 2)	(b) (- 2 , - 1)	(c) (2 , - 1)	(d) (- 2 , 0)
18)	Centre of the circle passes through the origin and the two points A (- 6 , 0) , B (0 , 8) is	(a) (4 , - 3)	(b) (- 5 , 5)	(c) (5 , 5)	(d) (- 3 , 4)
19)	How many circles whose centre (3 , - 5) and touches one of the two axes ?	(a) 1	(b) 2	(c) 3	(d) 4
20)	The point (2 , 2) lies the circle whose equation $x^2 + y^2 = 9$	(a) on	(b) inside	(c) outside	(d) in the centre of
21)	 The point (2 , 0) lies on	(a) x-axis.	(b) y-axis.	(c) the straight line $y = 2x$	(d) the circle $x^2 + y^2 = 9$
22)	 The point which lies on the circle : $(x - 2)^2 + y^2 = 13$ is	(a) (2 , 3)	(b) (3 , - 2)	(c) (2 , 5)	(d) (4 , 3)

23)

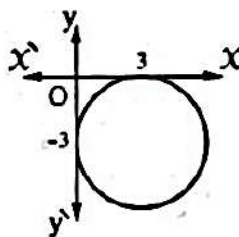
The circle $C : (x + 3)^2 + (y - 3)^2 = 9$ is represented by the figure



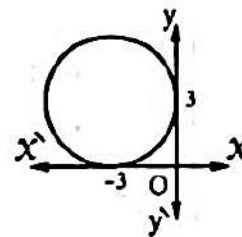
(a)



(b)



(c)



(d)

24)

The general form of the equation of a circle if its centre is $(2, -1)$ and its radius length = 3 cm. is

(a) $x^2 + y^2 - 4x + 2y - 9 = 0$

(b) $x^2 + y^2 - 4x + 2y - 4 = 0$

(c) $x^2 + y^2 + 2x - y + 3 = 0$

(d) $x^2 + y^2 + 2x - y + 9 = 0$

25)

The equation of the circle which is the image of the circle $x^2 + y^2 - 12x + 6y + 20 = 0$ by translation $(x + 2, y - 2)$ is

(a) $(x + 8)^2 + (y + 5)^2 = 25$

(b) $(x - 8)^2 + (y + 5)^2 = 25$

(c) $(x - 8)^2 + (y - 5)^2 = 25$

(d) $(x + 5)^2 + (y - 8)^2 = 25$

26)

The equation of the circle whose centre $(-4, 3)$ and passes through the origin point is

(a) $(x + 4)^2 + (y - 3)^2 = 5$

(b) $(x - 4)^2 + (y + 3)^2 = 25$

(c) $(x + 4)^2 + (y - 3)^2 = 625$

(d) $(x + 4)^2 + (y - 3)^2 = 25$

27)

The two circles $C_1 : (x + 2)^2 + (y - 1)^2 = 4$, $C_2 : (x - 5)^2 + (y - 3)^2 = 9$

(a) distant.

(b) touching externally.

(c) touching internally.

(d) intersecting.

28)

The intersection point of the circle $(X - 2)^2 + y^2 = 16$ with the X -axis is

- (a) $(6, 0), (-2, 0)$ (b) $(-6, 0), (2, 0)$
(c) $(4, 0), (-4, 0)$ (d) $(2, 0), (-2, 0)$

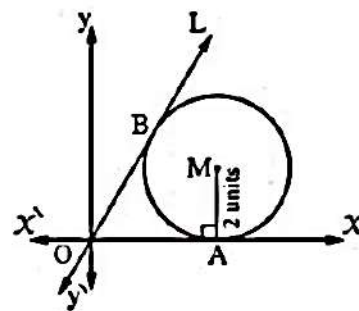
29)

In the opposite figure :

If $OB = 5$ length unit

, then the equation of the circle M is

- (a) $(X - 2)^2 + (y - 5)^2 = 25$
(b) $(X - 2)^2 + (y - 5)^2 = 4$
(c) $(X - 5)^2 + (y - 2)^2 = 25$
(d) $(X - 5)^2 + (y - 2)^2 = 4$



30)

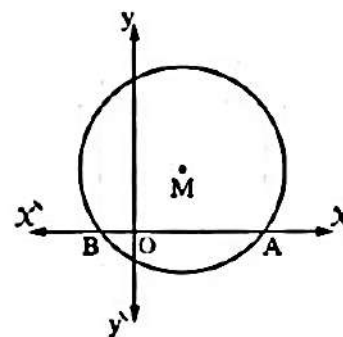
In the opposite figure :

If the equation of the circle is

$$(X - 2)^2 + (y - 3)^2 = 25$$

, then $AB = \dots\dots\dots$ length unit.

- (a) 8 (b) 4
(c) 6 (d) 5



31)

In the opposite figure :

If the equation of the circle M is

$$(X - 3)^2 + (y + 2)^2 = 25,$$

$\overline{AB}$ is a tangent to the circle M at A where

$B = (-2, 10)$, then $AB = \dots\dots\dots$ length unit.

- (a) 13 (b) $\sqrt{194}$ (c) 12 (d) 5

